ONLINE APPENDIX

Union and Firm Labor Market Power

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This Appendix is organized as follows. Section A shows the proofs of the propositions in the main text. Section B presents the reduced form bargaining. Section C presents details of our identification strategy. Section D provides additional tables and counterfactuals.

A Proofs

We first prove the following auxiliary Lemmas that are useful to prove Propositions 1 and 5.

Lemma 1. *Let $F_i'(L_i) \equiv \frac{dF_i}{dL_i} > 0$ and $F_i''(L_i) \equiv \frac{d^2F_i}{dL_i^2} < 0$. Then, the labor demand $\ell_i^D(w_i)$ is decreasing.*

Proof. We have that $w_i = F_i'(L_i)$. The labor demand is $\ell_i^D(w_i) = \left(F_i'\right)^{-1}(w_i)$. By the inverse function theorem we have $\frac{d\ell_i^D}{dw_i} = \left(F_i''(L_i)\right)^{-1} < 0$, as $F_i''(L_i) < 0$. □

Lemma 2. *When the labor demand constraint is binding, ψ_i is decreasing in w_i.*

Proof. When the labor demand is binding, the rationing share is defined implicitly by $\psi_i = \frac{\ell_i^D(w_i)}{\ell_i^S(\psi_i w_i)}$. Using the implicit function theorem and with a little abuse of notation we have

$$\frac{d\psi_i}{dw_i} = \left[\frac{\left(\ell_i^D\right)' \ell_i^S - \ell_i^D \left(\ell_i^S\right)' \psi_i}{\left(\ell_i^S\right)^2}\right]\left[1 + \frac{\ell_i^D \left(\ell_i^S\right)' w_i}{\left(\ell_i^S\right)^2}\right]^{-1},$$

where the first part in square brackets is negative as $\left(\ell_i^D\right)' < 0$ by Lemma 1 and $\left(\ell_i^S\right)' > 0$ by assumption. The second part in square brackets is positive. □

Lemma 3. *When the labor demand constraint is binding, and $-\xi(\ell_i^D, w_i) > 1$, then $\overline{w}_i$ is decreasing in w_i.*

Proof. When the labor demand is binding, the expected wage is defined implicitly by $\overline{w}_i = WB_i/\ell_i^S(\overline{w}_i)$, where $WB_i = w_i\ell_i^D(w_i)$ is the wage bill. Totally differentiating we get:

$$\frac{d\overline{w}_i}{dw_i} = \frac{\partial WB_i}{\partial w_i} + \frac{\partial \overline{w}_i}{\partial \ell_i^S} \cdot \frac{\partial \ell_i^S}{\partial \overline{w}_i} \cdot \frac{d\overline{w}_i}{dw_i}.$$

We have that $\frac{\partial \overline{w}_i}{\partial \ell_i^S} < 0$ and, by assumption $\frac{\partial \ell_i^S}{\partial \overline{w}_i} > 0$, so $\frac{\partial \overline{w}_i}{\partial \ell_i^S} \cdot \frac{\partial \ell_i^S}{\partial \overline{w}_i} < 0$. Solving for $\frac{d\overline{w}_i}{dw_i}$ we get:

$$\frac{d\overline{w}_i}{dw_i} = \frac{\partial WB_i}{\partial w_i}\left[1 - \frac{\partial \overline{w}_i}{\partial \ell_i^S} \cdot \frac{\partial \ell_i^S}{\partial \overline{w}_i}\right]^{-1}.$$

Then, the sign of $\frac{d\overline{w}_i}{dw_i}$ is determined by $\frac{\partial WB_i}{\partial w_i} = \left(\ell_i^D\right)' w_i + \ell_i^D$. This is negative whenever $-\left(\ell_i^D\right)' \frac{w_i}{\ell_i^D} > 1$, or what is the same, if the labor demand elasticity is greater than one. □

Proof of Proposition 1: Given a wage w_i and using $L_i = \psi_i\ell_i^S(\psi_i w_i)$, the employer solves:

$$\max_{\psi_i} F_i(\psi_i\ell_i^S(\psi_i w_i)) - w_i\psi_i\ell_i^S(\psi_i w_i)$$
$$\text{subject to:} \quad \psi_i \le 1.$$

The Lagrangian for this problem is:

$$\mathcal{L}_i(\psi_i, \zeta_i) = F_i(\psi_i\ell_i^S(\psi_i w_i)) - w_i\psi_i\ell_i^S(\psi_i w_i) - \zeta_i\left(\psi_i - 1\right).$$

With slight abuse of notation, the Kuhn-Tucker conditions are:

$$\left(F_i' - w_i\right)\left[\ell_i^S + w_i\psi_i(\ell_i^S)'\right] - \zeta_i = 0, \quad \psi_i \le 1, \quad \zeta_i \ge 0, \quad \text{and} \quad \zeta_i(\psi - 1) = 0.$$

Substituting the conditions with ζ_i and using $\left[\ell_i^S + w_i\psi_i(\ell_i^S)'\right] \ge 0$ gives us:

$$\left(F_i' - w_i\right) \ge 0, \quad \psi_i \le 1, \quad \text{and} \quad \left(F_i' - w_i\right)(\psi - 1) = 0.$$

By Lemma 1 we know the labor demand $\ell_i^D(w_i)$ is decreasing, so the first constraint becomes a labor demand constraint $\psi_i\ell_i^S(\psi_i w_i) \le \ell_i^D(w_i)$ For the last constraint, we have that $F_i' - w_i =$

$0 \iff \psi_i \ell_i^S(\psi_i w_i) = \ell_i^D(w_i)$. Therefore, we can rewrite the complementary slackness constraint as $(\psi_i - 1)(\psi_i \ell_i^S(\psi_i w_i) - \ell_i^D(w_i)) = 0$. This gives us all constraints for the wage bargaining stage: the labor supply constraint $\psi \leq 1$, the labor demand constraint, and the complementary slackness condition. As the labor supply only depends on w_i and ψ_i, any union objective function that originally depends on wages, employment and labor supply can be rewritten as a function of only w_i and ψ_i. $\square$

Proof of Proposition 2. Let $\overline{A}_i \equiv A_i \psi^{1-\delta}$. Aggregating employer output using (5), (7), the restriction $\frac{\beta_b}{1-\alpha_b} = 1 - \delta$, and that $L_i = \psi_i L_i^S$, the local labor market output is:

$$Y_m = \sum_{i \in \mathcal{I}_m} y_i = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} \overline{A}_i {L_i^S}^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} \overline{A}_i s_{i|m}^{1-\delta} {L_m^S}^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_m A_m {L_m^S}^{1-\delta},$$

where $L_m^S \equiv \sum_{i \in \mathcal{I}_m} L_i^S$. The local labor market productivity and misallocation are measured as:

$$\Omega_m \equiv \sum_{i \in \mathcal{I}_m} \frac{\overline{A}_i}{A_m} s_{i|m}^{1-\delta}, \quad A_m \equiv \sum_{i \in \mathcal{I}_m} A_i \tilde{s}_{i|m}^{1-\delta}, \quad \tilde{s}_{i|m} = \frac{\left(T_i^{1/\varepsilon_b} A_i\right)^{\varepsilon_b/1+\varepsilon_b \delta}}{\sum_{j \in \mathcal{I}_m} \left(T_j^{1/\varepsilon_b} A_j\right)^{\varepsilon_b/1+\varepsilon_b \delta}}.$$

$\tilde{s}_{i|m}$ comes from equation (16) with $\lambda_i = \psi_i = 1 \, \forall i \in \mathcal{I}$. Aggregating to sector level using (5):

$$Y_b = \sum_{m \in \mathcal{M}_b} Y_m = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{m \in \mathcal{M}_b} \Omega_m A_m {L_m^S}^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_b A_b {L_b^S}^{1-\delta}. \tag{A.1}$$

The sector level measures of productivity and misallocation are:

$$\Omega_b \equiv \sum_{m \in \mathcal{M}_b} \Omega_m A_m A_b^{-1} s_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} \overline{A}_i A_b^{-1} s_{i|m}^{1-\delta} s_{m|b}^{1-\delta},$$

$$A_b \equiv \sum_{m \in \mathcal{M}_b} A_m \tilde{s}_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} A_i \tilde{s}_{i|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta},$$

$$\tilde{s}_{m|b} = \frac{\left[\sum_{j \in \mathcal{I}_m} \left(T_j^{1/\varepsilon_b} A_j\right)^{\varepsilon_b/1+\varepsilon_b \delta}\right]^{\eta(1+\varepsilon_b \delta)/\varepsilon_b(1+\eta)}}{\sum_{m' \in \mathcal{M}_b} \left[\sum_{j' \in \mathcal{I}_{m'}} \left(T_{j'}^{1/\varepsilon_b} A_{j'}\right)^{\varepsilon_b/1+\varepsilon_b \delta}\right]^{\eta(1+\varepsilon_b \delta)/\varepsilon_b(1+\eta)}}.$$

From (3) and (7), the employer wage bill is: $w_i L_i = \beta_b \lambda_i P_b y_i$. Aggregating to m we get:

$$\sum_{i\in\mathcal{I}_m} w_i L_i = \beta_b \sum_{i\in\mathcal{I}_m} \lambda_i P_b y_i = \beta_b \sum_{i\in\mathcal{I}_m} \lambda_i \frac{P_b y_i}{P_b Y_m} P_b Y_m = \beta_b \lambda_m P_b Y_m, \text{ with } \lambda_m \equiv \sum_{i\in\mathcal{I}_m} \lambda_i \frac{\overline{A}_i}{\Omega_m A_m} s_{i|m}^{1-\delta}.$$

The market wedge λ_m is a value added weighted sum of λ_i. Aggregating to the sector level:

$$\sum_{m\in\mathcal{M}_b} \sum_{i\in\mathcal{I}_m} w_i L_i = \beta_b \sum_{m\in\mathcal{M}_b} \lambda_m \frac{P_b Y_m}{P_b Y_b} P_b Y_b = \beta_b \lambda_b P_b Y_b,$$
$$\lambda_b \equiv \sum_{m\in\mathcal{M}_b} \lambda_m \frac{A_m \Omega_m}{\Omega_b A_b} s_{m|b}^{1-\delta} = \sum_{m\in\mathcal{M}_b} \sum_{i\in\mathcal{I}_m} \lambda_i \frac{\overline{A}_i}{\Omega_b A_b} s_{i|m}^{1-\delta} s_{m|b}^{1-\delta}.$$

Let $\widetilde{\widetilde{w}}_i$ be the market normalized wages $\widetilde{w}_i$ as defined by (A.9), but using $\overline{A}_i$ instead of A_i. Then, we have that the expected wage is: $\overline{w}_i = \widetilde{\widetilde{w}}_i \Phi_m^{(1-\eta/\varepsilon_b)\delta/1+\varepsilon_b\delta} P_b^{1/(1-\alpha_b)(1+\varepsilon_b\delta)} \left(\frac{\Phi}{L}\right)^{\delta/1+\varepsilon_b\delta}$. Using the definition of $\Phi_m \equiv \sum_{i\in\mathcal{I}_m} T_i \overline{w}_i^{\varepsilon_b}$:

$$\Phi_m = \widetilde{\Phi}_m^{1+\varepsilon_b\delta/1+\eta\delta} P_b^{\varepsilon_b/(1-\alpha_b)(1+\eta\delta)} \left(\frac{\Phi}{L}\right)^{\varepsilon_b\delta/1+\eta\delta}, \quad \widetilde{\Phi}_m \equiv \sum_{i\in\mathcal{I}_m} T_i \widetilde{\widetilde{w}}_i^{\varepsilon_b}, \tag{A.2}$$

where $\widetilde{\Phi}_m$ is then a function of labor supply and rationing shares $\{s_{i|m}, \psi_i\}_{i\in\mathcal{I}_m}$. Substituting Φ_m into the expected wage expression above we get:

$$\overline{w}_i = \widetilde{\widetilde{w}}_i \widetilde{\Phi}_m^{\frac{(\varepsilon_b-\eta)\delta}{\varepsilon_b(1+\eta\delta)}} P_b^{\frac{1}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\eta\delta}}.$$

The employer labor supply is $L_i^S = s_{i|m} s_{m|b} L_b^S$. Given the normalized wages per market $\{\widetilde{\widetilde{w}}_i\}$, we can compute the labor supply shares within the local labor market and the share of m out of the sector using the definition of $\Phi_b \equiv \sum_{m\in\mathcal{M}_b} \Phi_m^{\eta/\varepsilon_b}$ and (A.2):

$$s_{i|m} = \frac{T_i \overline{w}_i^{\varepsilon_b}}{\Phi_m} = \frac{T_i \widetilde{\widetilde{w}}_i^{\varepsilon_b}}{\widetilde{\Phi}_m}, \quad s_{m|b} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_b} = \frac{\widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta\delta)}}}{\widetilde{\Phi}_b}, \quad \text{with} \quad \widetilde{\Phi}_b \equiv \sum_{m\in\mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta\delta)}},$$

where $\mathcal{M}_b$ is the set of all local labor markets in b. Using (A.2), sector labor supply is a function of prices and sector 'tilde' variables $\widetilde{\Phi}_b(\mathbf{s}_b, \mathbf{\Psi}_b)$, where $\mathbf{s}_b \equiv \{s_{i|m}\}_{i\in\mathcal{I}_b}$ and $\mathbf{\Psi}_b \equiv \{\psi_i\}_{i\in\mathcal{I}_b}$:

$$L_b^S = \frac{\Phi_b \Gamma_b^\eta}{\sum_{b'\in\mathcal{B}} \Phi_{b'} \Gamma_{b'}^\eta} L^S = \frac{P_b^{\eta/(1-\alpha_b)(1+\eta\delta)} \widetilde{\Phi}_b(\mathbf{s}_b, \mathbf{\Psi}_b) \Gamma_b^\eta}{\widetilde{\Phi}} L^S, \quad \widetilde{\Phi} \equiv \sum_{b'\in\mathcal{B}} P_{b'}^{\eta/(1-\alpha_{b'})(1+\eta\delta)} \widetilde{\Phi}_{b'}(\mathbf{s}_{b'}, \mathbf{\Psi}_{b'}) \Gamma_{b'}^\eta. \tag{A.3}$$

This is where the simplifying assumption $1-\delta = \beta_b (1-\alpha_b)^{-1}$ buys us tractability. We can factor out the economy wide constant Φ from (A.2) and leave everything in terms of labor supply shares, rationing shares, and prices. □

Proof of Proposition 3. Using the intermediate sector output demand (6), the expression for intermediate output (A.1) and the aggregate production function from (5) we get the desired expression for each sector b:

$$P_b^{\frac{1}{1-\alpha_b}} A_b \Omega_b L_b^S(\mathbf{P})^{1-\delta} = \theta_b \prod_{b'\in\mathcal{B}} \left[P_{b'}^{\frac{\alpha_{b'}}{1-\alpha_{b'}}} A_{b'} \Omega_b L_{b'}^S(\mathbf{P})^{1-\delta} \right]^{\theta_{b'}}. \tag{A.4}$$

Proof of Proposition 4. Substituting (A.3) into (A.4) we get:

$$P_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}} A_b \Omega_b \left(\widetilde{\Phi}_b \Gamma_b^\eta\right)^{1-\delta} = \theta_b \prod_{b'\in\mathcal{B}} \left(A_{b'} \Omega_{b'} \left(\widetilde{\Phi}_{b'} \Gamma_{b'}^\eta\right)^{1-\delta} \right)^{\theta_{b'}} \prod_{b'\in\mathcal{B}} \left(P_{b'}^{\frac{\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}} \right)^{\theta_{b'}}. \tag{A.5}$$

Define $f_b \equiv (1-\alpha_b)^{-1} \log(P_b)$ and $\mathbf{f}$ as a $B\times 1$ vector whose element b' is $f_{b'}$. Then, taking logs and rearranging the previous expressions for all $b \in \mathcal{B}$ we obtain:

$$\mathbf{f} = \mathbf{C} + \mathbf{D}\mathbf{f}, \tag{A.6}$$

where $\mathbf{C}$ is a $B\times 1$ vector whose b element is

$$(\mathbf{C})_b = \frac{1+\eta\delta}{1+\eta} \left[\log\left(\frac{\theta_b}{A_b\Omega_b}\right) - (1-\delta)\log\left(\widetilde{\Phi}_b\Gamma_b^\eta\right) + \sum_{b'\in\mathcal{B}} \theta_{b'} \left(\log(A_{b'}\Omega_{b'}) + (1-\delta)\log(\widetilde{\Phi}_{b'}\Gamma_{b'}^\eta) \right) \right],$$

and $\mathbf{D}$ is a $B\times B$ matrix whose b row b' column element is:

$$(\mathbf{D})_{bb'} = \frac{(\alpha_{b'}(1+\eta\delta) + \eta(1-\delta))\,\theta_{b'}}{1+\eta}.$$

A solution to the system (A.6) exists and is unique if the matrix $\mathbf{I}-\mathbf{D}$ is invertible. This matrix has an eigenvalue of zero, and therefore is not invertible, if and only if $\mathbf{D}$ has a unit eigenvalue.[1] The matrix $\mathbf{D}$ has a unit eigenvalue if and only if the sum of the elements of the rows in matrix $\mathbf{D}$ are equal to 1. To see this, let $\mathbf{v}$ be the eigenvector associated with the unit eigenvalue of $\mathbf{D}$, i.e. $\mathbf{Dv}=\mathbf{v}$. If $\mathbf{v}=\mathbf{1}$, then, by the Perron-Frobenius theorem, it is the only eigenvector (up-to-scale) associated with the unit eigenvalue. Furthermore, if $\mathbf{v}=\mathbf{1}$, then $\sum_{b'}(D)_{bb'}=1$ for all $b\in\mathcal{B}$. Conversely, if $\sum_{b'}(D)_{bb'}=1$ for all $b\in\mathcal{B}$, then $\mathbf{v}=\mathbf{1}$ is a solution for the eigensystem $\mathbf{Dv}=\mathbf{v}$. But, by the Perron-Frobenius theorem, $\mathbf{v}=\mathbf{1}$ is the unique (up-to-scale) eigenvector associated with the unit eigenvalue. Therefore, the matrix $\mathbf{I}-\mathbf{D}$ is not invertible if and only if the sum of the elements of the rows in matrix $\mathbf{D}$ are equal to 1.

This sum is equal to 1 if and only if $\sum_b \alpha_b\theta_b=1$ as:

$$\begin{aligned}\sum_{b'}(\mathbf{D})_{bb'}=1 \iff & \sum_{b'}\left(\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)\right)\theta_{b'}=1+\eta \\ \iff & \sum_{b'}\alpha_{b'}\theta_{b'}=\frac{1+\eta-\eta(1-\delta)}{1+\eta\delta} \iff \sum_{b}\alpha_{b'}\theta'_{b}=1.\end{aligned}$$

Therefore whenever $\sum_b\alpha_b\theta_b\neq 1$, $\mathbf{f}$ has a unique solution. As, $\alpha_b<1$ for all $b\in\mathcal{B}$ and $\sum_b\theta_b=1$, then the vector of prices $\mathbf{P}$ has a unique solution. Solving for P_b in (A.5):

$$P_b=X_bX^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}},\ X_b=\left(\frac{\theta_b}{A_b\Omega_b(\widetilde{\Phi}_b\Gamma_b^{\eta})^{(1-\delta)}}\right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}},\ X=\left(\prod_{b'\in\mathcal{B}}\left(\frac{\theta_{b'}}{X_{b'}}\right)^{\theta_{b'}}\right)^{\frac{1+\eta}{(1+\eta\delta)\sum_{b'\in\mathcal{B}}\theta_{b'}(1-\alpha_{b'})}}, \tag{A.7}$$

for all $b\in\mathcal{B}$ where we used the aggregate price index $1=\prod_{b\in\mathcal{B}}\left(\frac{P_b}{\theta_b}\right)^{\theta_b}$ to find the economy wide constant X. The above is the closed-form solution for prices. □

Proof of Proposition 5: The only situation where $\psi_i<1$ is when the labor demand is binding and the labor supply constraint is slack. The labor demand is derived such that for a given wage, the firm chooses an employment to maximize profits. Then, the profit function becomes $F_i\left(\ell_i^D(w_i)\right)-w_i\ell_i^D(w_i)$, which is clearly decreasing in w_i. To see this, fix ℓ_i^D and decrease w_i.

[1]Proof: If 1 is an eigenvalue of $\mathbf{D}$, then $\mathbf{Dv}=\mathbf{v}$ for a nonzero vector $\mathbf{v}$. Then $(\mathbf{I}-\mathbf{D})\mathbf{v}=0$, so 0 is an eigenvalue of $\mathbf{I}-\mathbf{D}$ with the associated eigenvector $\mathbf{v}$. Conversely, if 0 is an eigenvalue of $\mathbf{I}-\mathbf{D}$, then $\mathbf{Dv}=\mathbf{v}$ and 1 is an eigenvalue of $\mathbf{D}$.

Clearly the profit increases. Then as, $\ell_i^D(w_i)$ is defined such that profit is maximized, once employment is adjusted the profit should (weakly) increase from the fixed employment situation. Now, by assumption the union's objective function can be written as an increasing function of $\overline{w}_i$ and ψ_i. By Lemmas 2 and 3, ψ_i and $\overline{w}_i$ are decreasing functions of w_i when the labor demand is binding. So, whenever the labor demand is binding, the Nash product is maximized by decreasing as much as possible the wage. This would hit the labor supply constraint, giving us $\psi_i = 1$. □

Proof of Proposition 6: If $G_i(w_i) \equiv G_{i,1}(w_i) - G_{i,0}$, with $G_{i,1}(w_i) \geq G_{i,0}$, then $\xi(G_i, w_i) = \xi(G_{i,1}, w_i)\frac{G_{i,1}(w_i)}{G_{i,1}(w_i)-G_{i,0}}$. Then clearly $\frac{\partial\xi(G_i,w_i)}{\partial G_{i,0}} > 0$ and $\frac{\partial \omega_i}{\partial G_{i,0}} > 0$. □

Proof of Proposition 7. If $\psi_i = 1$, $L_i = L_i^S$. Using $\frac{\beta_b}{1-\alpha_b} = 1-\delta$, plugging (8) into (11) we get:

$$w_i = \left(\beta_b \lambda_i \frac{A_i}{T_i^{\delta}}\right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi_m^{\frac{\delta(1-\eta/\varepsilon_b)}{1+\varepsilon_b\delta}} P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \left(\frac{\Phi}{\Gamma_b^{\eta} L^S}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}. \tag{A.8}$$

From (8), the labor supply share of employer i is:

$$s_{i|m} = \frac{T_i w_i^{\varepsilon_b}}{\sum_{j\in\mathcal{I}_m} T_j w_j^{\varepsilon_b}} = \frac{\left(T_i^{\frac{1}{\varepsilon_b}} A_i \lambda_i\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\sum_{j\in\mathcal{I}_m}\left(T_j^{\frac{1}{\varepsilon_b}} A_j \lambda_j,\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}},$$

where we used equation (A.8) in the second equality. □

Proof of Proposition 8. **Existence**. The function defined by (16) maps from the set $[0,1]^{N_m}$ to itself. Then, by Brouwer's fixed point theorem at least one solution exists. □

Uniqueness. In the Supplemental Material we present a Theorem and a Corollary from Allen et al. (2016) that we use to establish uniqueness.

We first prove uniqueness of a transformed system using *sector* normalized wages x_i which have a one-to-one correspondence with the labor supply shares. Then, using Lemma we show that uniqueness of the system defined with x_i's imply the uniqueness of *market* normalized wages $\widetilde{w}_i$, which are the ones we use in the computer and later for aggregation.

Using $\Phi_m = T_i w_i^{\varepsilon_b}(s_{i|m})^{-1}$, solving for w_i and dividing by the common sector variables we get the *sector* normalized wages x_i:

$$x_i = \left(\beta_b \min\{\widetilde{\lambda}_i, 1\} A_i \left(T_i^{\frac{1}{\varepsilon_b}} \Gamma_b\right)^{-\delta\eta}\right)^{\frac{1}{1+\eta\delta}} \left(s_{i|m}\right)^{\frac{\delta(\eta-\varepsilon_b)}{\varepsilon_b(1+\eta\delta)}}.$$

Clearly, the labor supply shares can be only a function of the vector of normalized wages $\mathbf{x}$ instead of the vector of wages. Then we have a system of the form $\mathbf{x} = f_i(\mathbf{x})$, where there is a one-to-one mapping from the vector of normalized wages $\mathbf{x}$ to labor supply shares.

Define $g : \mathbb{R}^{N_m}_{++} \to \mathbb{R}^{N_m}$:

$$g_i(\mathbf{x}) = f_i(\mathbf{x}) - x_i, \quad \forall i \in \{1, .., N_m\}.$$

We want to prove that the solution satisfying $g(\mathbf{x}) = 0$ is unique. In order to do so, we first need to show that $g(\mathbf{x})$ satisfies the gross substitution property ($\frac{\partial g_i}{\partial x_j} > 0$ for any $j \neq i$).

Taking the partial derivative of g_i with respect to x_j for any $j \neq i$:

$$\frac{\partial g_i}{\partial w_j} = \underbrace{\frac{\partial f_i(\mathbf{x})}{\partial \widetilde{\lambda}_i}}_{\geq 0} \times \underbrace{\frac{\partial \widetilde{\lambda}_i}{\partial s_{i|m}}}_{<0} \times \underbrace{\frac{\partial s_{i|m}}{\partial x_j}}_{<0} + \underbrace{\frac{\partial f_i(\mathbf{x})}{\partial s_{i|m}}}_{<0} \underbrace{\frac{\partial s_{i|m}}{\partial x_j}}_{<0}.$$

Therefore $\frac{\partial g_i}{\partial w_j} > 0$ for any $i \neq j$ and g satisfies the gross-substitution property required by Theorem 1 in the Supplemental Material.

The remaining condition to prove to use Corollary 1 in the Supplemental Material is simply that $f_i(\mathbf{w})$ is homogeneous of a degree smaller than 1. Clearly, $f_i(\mathbf{w})$ is homogeneous of degree 0 as labor supply share and the wedge are homogeneous of degree zero. Therefore, the function g satisfies the conditions of Corollary 1, and we conclude that there exists at most one solution satisfying $g(\mathbf{x}) = 0$. As $\mathbf{x}$ is a bijective function of labor supply shares, we can conclude that the original system also has a unique solution.

When solving the model in the computer we solve for market level normalized wages:

$$\widetilde{w}_i = \left(\beta_b \min\{\widetilde{\lambda}_i, 1\} \frac{A_i}{\left(T_i \Gamma_b^{\eta}\right)^{\delta}}\right)^{\frac{1}{1+\varepsilon_b\delta}}. \tag{A.9}$$

There is a one-to-one mapping from the sector normalized and market normalized wages, so the solution of the system formed by market normalized wages above is also unique. □

B Reduced-form bargaining model

We can rearrange the decomposition (13) to define the union's rent:

$$\underbrace{w_i L_i - \mu_i \nu_i F_i(L_i)}_{\text{Union's Rent}} = \omega_i \overbrace{\left[F_i(L_i) \quad - \quad \underbrace{\mu_i \nu_i F_i(L_i)}_{\text{Wage bill in oligopsony}} \right]}^{\text{Total Rents}}.$$

This shows that any payment above the oligopsony wage bill—the union's rent—equals a fraction ω_i of the total rents, which equal revenue minus the oligopsony wage bill. The employer's profit equals the remaining fraction $(1-\omega_i)$ of these total rents. This means that, after employment is determined (L_i is fixed) and oligopsony wages are paid, we can characterize wages *as if* the union and employer solve this reduced-form Nash bargaining problem:

$$\max_{w_i} \left(w_i L_i - \mu_i \nu_i F_i(L_i) \right)^{\omega_i} \left(F_i(L_i) - w_i L_i \right)^{1-\omega_i},$$

where ω_i represents the bargaining powers in this reduced form. This formulation clarifies how ω_i summarizes the union market power by determining the split of total rents. Taking ω_i and μ_i as exogenous in this problem, the first order conditions are:

$$w_i = \omega_i \frac{F_i(L_i)}{L_i} + (1-\omega_i)\mu_i \nu_i \frac{F_i(L_i)}{L_i} = \left[\omega_i \frac{1}{\nu_i} + (1-\omega_i)\mu_i \right] F'(L_i),$$

where we used that $\nu_i \equiv F'(L_i)\frac{L_i}{F(L_i)}$. The solution to this bargaining problem is therefore the same to (3) with $\lambda_i < 1$.

C Identification and estimation

Here we first show additional details for the identification through heteroskedasticity of sector common parameters η and δ, including the proof of Proposition 9. Second, we discuss why our method to identify the within-market elasticities of substitution is still valid even in the presence of strategic interactions. Third, we show how to identify amenities. Fourth, we explain how to identify counterfactuals using exact-hat algebra. Finally, we show how the model hits other non-targeted moments.

C.1 Details on identification of common parameters η and δ

Define $\widetilde{\delta} \equiv -\delta$. Then, the reduced form solution for the system (17)-(18) is:

$$\ln w_i = \frac{1}{1-\widetilde{\delta}\eta}\left(\ln A_i + \widetilde{\delta}\ln \widetilde{T}_i\right) \quad \text{and} \quad \ln L_i = \frac{1}{1-\widetilde{\delta}\eta}\left(\eta \ln A_i + \ln \widetilde{T}_i\right). \tag{C.1}$$

Denote $\sigma_{w,o}$ and $\sigma_{L,o}$ as the variances of wages and employment for occupation o. Also, let $\sigma_{wL,o}$ be the covariance between wages and employment for occupation o. Furthermore, let $\Delta_{w,12} = \sigma_{w,1} - \sigma_{w,2}$ be the difference in wage variances between occupations 1 and 2. Define analogously $\Delta_{L,12}$ for employment variances and $\Delta_{wL,12}$ for covariances. The same notation applies for differences between occupations 3 and 4.

After some algebraic manipulation, the moment conditions $\mathbb{E}\left(\ln A_{i,1} \ln \widetilde{T}_{i,1}\right) = \mathbb{E}\left(\ln A_{i,2} \ln \widetilde{T}_{i,2}\right)$ and $\mathbb{E}\left(\ln A_{i,3} \ln \widetilde{T}_{i,3}\right) = \mathbb{E}\left(\ln A_{i,4} \ln \widetilde{T}_{i,4}\right)$ yield the following equations, respectively:

$$\eta\Delta_{w,12} + \widetilde{\delta}\Delta_{L,12} = (1+\widetilde{\delta}\eta)\Delta_{wL,12}, \quad \text{and} \quad \eta\Delta_{w,34} + \widetilde{\delta}\Delta_{L,34} = (1+\widetilde{\delta}\eta)\Delta_{wL,34}. \tag{C.2}$$

Each of these equations represents a hyperbola in the $(\widetilde{\delta}, \eta)$ plane. Proposition 9 establishes conditions for the existence of a solution to the system (C.2).

Proof of Proposition 9. Define $\widetilde{\theta}_1 = \frac{\eta}{1+\widetilde{\delta}\eta}$ and $\widetilde{\theta}_2 = \frac{\widetilde{\delta}}{1+\widetilde{\delta}\eta}$. We can rewrite the system (C.2) as:

$$\begin{pmatrix} \Delta_{w,12} & \Delta_{L,12} \\ \Delta_{w,34} & \Delta_{L,34} \end{pmatrix} \begin{pmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \end{pmatrix} = \begin{pmatrix} \Delta_{wL,12} \\ \Delta_{wL,34} \end{pmatrix}. \tag{C.3}$$

Note that by rewriting the system in this way we make the assumption that $\widetilde{\delta}\eta \neq -1$. This is without loss of generality as we could also rewrite the system by normalizing by either η or $\widetilde{\delta}$ and assuming they are nonzero. Regardless of the normalization, we get the same result. We keep this one for expositional clarity.

Given $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$, there are two solutions for η and $\widetilde{\delta}$. In fact, it is easy to show that if $(\eta, \widetilde{\delta})$ is a solution then $(1/\widetilde{\delta}, 1/\eta)$ is also a solution. If the two solutions yield parameters with different signs, the sign restrictions on η and $\widetilde{\delta}$ identify the relevant solution.

What is left to show is that the system (C.3) has a solution. This is equivalent to showing that the matrix has full rank, or equivalently, its determinant is not zero:

$$\Delta_{w,12}\Delta_{L,34} - \Delta_{w,34}\Delta_{L,12} \neq 0. \tag{C.4}$$

From the reduced form solution (C.1) we have that

$$\Delta_{w,12} = \frac{1}{\left(1-\widetilde{\delta}\eta\right)^2}\left(\Delta_{A,12} + \widetilde{\delta}^2\Delta_{T,12}\right) \quad \text{and} \quad \Delta_{L,12} = \frac{1}{\left(1-\widetilde{\delta}\eta\right)^2}\left(\eta^2\Delta_{A,12} + \Delta_{T,12}\right),$$

where $\Delta_{A,12} = \varsigma_{A,1} - \varsigma_{A,2}$ and $\Delta_{T,12} = \varsigma_{T,1} - \varsigma_{T,2}$. Analogous expressions for (3,4) follow. Substituting these expressions into (C.4), after some simplifications we get:

$$\left(1-\widetilde{\delta}\eta\right)\left(1+\widetilde{\delta}\eta\right)\left(\Delta_{A,12}\Delta_{T,34} - \Delta_{T,12}\Delta_{A,34}\right) \neq 0.$$

As $\widetilde{\delta}\eta \neq 1$ by assumption on the signs of η and δ, and $\widetilde{\delta}\eta \neq -1$ by our normalization assumption when defining $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$, we only need $(\Delta_{A,12}\Delta_{T,34} - \Delta_{T,12}\Delta_{A,34}) \neq 0$, which is precisely the rank condition in the proposition. □

C.2 Elasticity of substitution and labor supply elasticity with strategic interactions.

Berger et al. (2022) (BHM) show that *within-establishment, across-time* variation cannot identify the labor supply elasticity because non-atomistic establishments' strategic interactions can affect the overall equilibrium, resulting in a SUTVA violation. We expand on BHM's argument in three ways: (i) we clarify the general relationship between the elasticity of substitution and the labor supply elasticity and explain the scenarios where they are equivalent; (ii) we establish generally the bias between the labor supply elasticity and a reduced form estimate; and (iii) we show that within-equilibrium variation can identify the local elasticity of substitution.

Our method avoids the identification issues raised by BHM for identifying supply or demand elasticities under strategic interactions. Paraphrasing BHM, the labor supply elasticity asks the following question: *how much would employment change within a firm after increasing its wage by one percent and holding the other firms' response constant?* Thus, the supply elasticity is a partial equilibrium object: $\left.\frac{d\ln L_i}{d\ln w_i}\right|_{w_{-i}}$.

BHM argue that even when there is a well-identified idiosyncratic demand shock and no labor supply shifters, we cannot identify the firm's labor supply elasticity. This is because the strategic interactions of other market participants will change the firm's labor supply curve after the shock has occurred. This change in the equilibrium allocation violates the stable unit treatment value assumption (SUTVA). Then, by using within-firm across-equilibrium variation in a reduce-form exercise we are measuring $\frac{d\ln L_i}{d\ln w_i}$ rather than $\left.\frac{d\ln L_i}{d\ln w_i}\right|_{w_{-i}}$.

Consider the following decomposition of the reduced-form estimate:[2]

$$\frac{d\ln L_i}{d\ln w_i} = \frac{d\ln\left(L_i/L_j\right)}{d\ln\left(w_i/w_j\right)}\left(1-\frac{d\ln w_j}{d\ln w_i}\right)+\frac{d\ln L_j}{d\ln w_i}, \tag{C.5}$$

where L_j and w_j are the employment and wages for any other j within m. In our setup, the

[2] $\frac{d\ln L_i}{d\ln w_i} = \frac{d\ln L_i}{d\ln\left(w_i/w_j\right)}\frac{d\ln\left(w_i/w_j\right)}{d\ln w_i} = \frac{d\left(\ln L_i-\ln L_j+\ln L_j\right)}{d\ln\left(w_i/w_j\right)}\frac{d\left(\ln w_i-\ln w_j\right)}{d\ln w_i} = \frac{d\ln\left(L_i/L_j\right)}{d\ln\left(w_i/w_j\right)}\left(1-\frac{d\ln w_j}{d\ln w_i}\right)+\frac{d\ln L_j}{d\ln w_i}$.

elasticity of substitution $\frac{d\ln\left(L_i/L_j\right)}{d\ln\left(w_i/w_j\right)}$ is equal to ε_b. The structural labor supply elasticity is:

$$\left.\frac{d\ln L_i}{d\ln w_i}\right|_{w_{-i}} = \varepsilon_b + \underbrace{\left.\frac{d\ln L_j}{d\ln w_i}\right|_{w_{-i}}}_{\text{Cross-elasticity}} = \varepsilon_b(1-s_i) + \eta s_i,$$

where the cross-elasticity is equal to $-\varepsilon_b s_i + \eta s_i$ given our Bertrand competition environment.

The relation between the reduced form estimate and the labor supply elasticity is:

$$\underbrace{\frac{d\ln L_i}{d\ln w_i}}_{\text{Reduced-form}} = \underbrace{\left.\frac{d\ln L_i}{d\ln w_i}\right|_{w_{-i}}}_{\text{Supply elasticity}} + \underbrace{\left(\frac{d\ln L_j}{d\ln w_i} - \left.\frac{d\ln L_j}{d\ln w_i}\right|_{w_{-i}}\right) - \varepsilon_b \frac{d\ln w_j}{d\ln w_i}}_{\text{Bias}}.$$

The reduced-form estimate is equal to the labor supply elasticity in two cases. First, when the establishment is atomistic because other firms in the market do not respond, so $\frac{d\ln w_j}{d\ln w_i} = 0$. Second, when the local and across-market elasticity of substitution are the same. In such case, the employment loss of the competitors is completely offset by the increase in employment to the local labor market. Then, the cross-elasticity is zero, and $\frac{d\ln L_j}{d\ln w_i} - \varepsilon_b \frac{d\ln w_j}{d\ln w_i} = 0$. In both cases, the labor supply elasticity is equal to the local elasticity of substitution.[3]

BHM use the relation between the reduced-form estimate and the structural elasticity to indirectly infer the structural parameters ε_b and η. However, this requires assuming a wage-setting process to pin down the structural elasticity. In contrast, our method identifies both parameters without making any assumptions about the wage-setting process.

Consider Figure C.1 panel A to illustrate the argument. Assume there is no bargaining. The structural labor supply elasticity measures the employment response to a wage increase along a labor supply curve L_i^S. If the firm is not atomistic, the wage increase after a positive idiosyncratic productivity shock will affect the other firms in the market, leading to a shift in establishment's L_i^S. Under Bertrand competition, wages are strategic complements, resulting in an upward shift of L_i^S, and the reduced-form elasticity will be smaller than the structural one. Under Cournot competition, employment levels are strategic substitutes, resulting in a downward shift and the reduced-form elasticity being greater than the structural one.[4]

[3]There is a third trivial case when the firm is the only one in the market and the supply elasticity is η.

[4]The labor supply elasticity in the Cournot competition case is given by $\left(\frac{1}{\varepsilon_b}(1-s_i) + \frac{1}{\eta}s_i\right)^{-1}$.

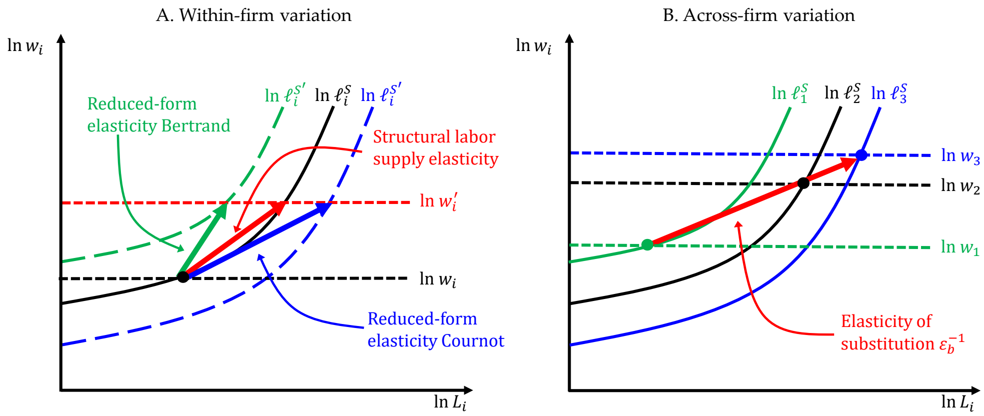


Figure C.1: Within-firm across-equilibria vs Across-firm within-equilibrium variation

Notes: Panel A: reduced-form and structural labor supply elasticities following an idiosyncratic shock depending on the market competition. Panel B: variation across employment and wages within m identify ε_b.

A regression of log employment on log wages within a local labor market *conditional* on an equilibrium allocation identifies ε_b. Consider Figure C.1 panel B. We have three different employers that only differ in their productivities. As they are not atomistic, the labor supply intercepts—which are a function of the competitors wages—are different.[5] For any two establishments i, j, the slope of the straight line connecting two points in the log wage and employment plane is $\frac{\ln w_i - \ln w_j}{\ln L_i - \ln L_j} = \varepsilon_b^{-1}$. Since we are conditioning on an equilibrium allocation, SUTVA is not violated, and the slope estimate of regressing log employment on log wages must be equal to ε_b.[6] This regression estimates the local elasticity of substitution.

In conclusion, the source of BHM's identification problem comes from not controlling for equilibrium changes upon idiosyncratic shocks. Introducing market-year fixed effects solves the problem.[7]

C.3 Amenities

We recover employer amenities so that in equilibrium the wages and labor allocations are the same as in the data. Using the labor supply (8), we back out amenities up to a constant:

[5] The log inverse labor supply is equal to $\log w_i = \frac{1}{\varepsilon} \log \left(\frac{L_i}{L_m - L_i} \right) + \frac{1}{\varepsilon} \log \left(\sum_{j \neq i} w_j \right)$.

[6] Consider a regression: $\ln L_i = b_0 + b_1 \ln w_i$ without supply side shifters so we do not include an error term. Demeaning both $\ln L_i$ and $\ln w_i$ and regressing those without a constant term we get the estimate of $b_1 = \frac{d\left[\ln(L_i) - \frac{1}{N}\sum_j \ln(L_j)\right]}{d\left[\ln(w_i) - \frac{1}{N}\sum_j \ln(w_j)\right]} = \frac{d \ln\left(L_i \big/ \left(\prod_j L_j\right)^{1/N}\right)}{d \ln\left(w_i \big/ \left(\prod_j w_j\right)^{1/N}\right)} = \varepsilon_b$.

[7] The argument extends to regressions in differences changing the interpretation of the fixed effects.

$T_i = \frac{s_{i|m}}{w_i^{\varepsilon_b}}\Phi_m$. To see that, write the labor supply share of market m with relative to an arbitrary market 1: $\frac{L_m^S}{L_1^S} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_1^{\eta/\varepsilon_{b'}}}\frac{\Gamma_b^\eta}{\Gamma_1^\eta}$. The local labor market aggregate is then: $\Phi_m = \left(\frac{L_m^S}{\Gamma_b^\eta}\frac{\Gamma_1^\eta}{L_1^S}\Phi_1^{\eta/\varepsilon_{b'}}\right)^{\varepsilon_b/\eta}$. We normalize $\frac{\Gamma_1^\eta}{L_1^S}\Phi_1^{\eta/\varepsilon_{b'}} = 1$. Substituting into the amenity, we get: $T_i = \frac{s_{i|m}}{(w_i)^{\varepsilon_b}}\left(\frac{L_m^S}{\Gamma_b^\eta}\right)^{\varepsilon_b/\eta}$.

C.4 Hat algebra

Here we show how to compute the counterfactuals in general equilibrium by using revenue productivities (TFPRs), which are a function of prices determined in general equilibrium, and not just the underlying physical productivities. A priori, the issue is that counterfactually changing the labor wedge changes equilibrium prices and therefore the TFPRs.

The literature on misallocation has used the TFPRs, together with a modeling assumption on the sector price, to compute the normalized within sector productivity distribution. This has prevented performing general equilibrium counterfactuals that also take into account productivity differences across industries.[8] We show that we can: (i) do counterfactuals in general equilibrium by writing the model relative to a baseline scenario; and (ii) compute the movement of production factors across sectors.

We write counterfactual sector prices relative to the baseline and to fix the transformed revenue productivities Z_i.[9] From the definition of $Z_i = P_b^{\frac{1}{1-\alpha_b}} A_i$ and equation (11), wages are: $w_i = \beta_b \lambda_i Z_i L_i^{-\delta}$. Denoting with a prime the variables in the counterfactual (e.g. P_b') and with a hat the relative variables (e.g. $\widehat{P}_b = P_b'/P_b$). The counterfactual revenue productivity is a function of the relative price $\widehat{P}_b$ and the observed revenue productivity Z_i.

Let λ_i' be the counterfactual wedge, and ψ_i' the counterfactual rationing share. We have three cases. When employer i is: (i) only supply constrained, $\psi_i' = 1$ and $\lambda_i' < 1$; (ii) only demand constrained, $\psi_i' < 1$ and $\lambda_i' = 1$; and (iii) demand and supply constrained, $\psi_i' = 1$ and $\lambda_i' = 1$.

[8]For example, Hsieh and Klenow (2009) conduct a counterfactual where they remove distortions at the firm level and compute the productivity gains at the sector level from production reallocating to more productive firms *within* each sector. This allows them to compute a *partial* equilibrium effect on total factor productivity as they can identify only relative productivity differences within each sector while normalizing average differences across industries. For more details, see equation (19) and the discussion below in their paper.

[9]Solving the counterfactuals in levels requires making some additional normalizations. One could assume that the minimum physical productivity (TFP) is constant across sectors normalize them to get rid of P_b.

The counterfactual expected wage for any i is:

$$\overline{w}_i' = \beta_b \lambda_i' Z_i \psi_i'^{1-\delta} \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{S\prime -\delta} = \beta_b \lambda_i' \overline{Z}_i' \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{S\prime -\delta},$$

where Z_i is taken as a fixed fundamental and $\overline{Z}_i' \equiv Z_i \psi_i'^{1-\delta}$ is endogenous. We have to solve for sector prices relative to the baseline $\widehat{P}_b$. Substituting the labor supply in the expression above, we get a similar expression to (A.8) for counterfactual expected wages:

$$\overline{w}_i' = \left(\frac{\beta_b \lambda_i' \overline{Z}_i'}{(T_i \Gamma_b^{\eta})^{\delta}} \right)^{\frac{1}{1+\varepsilon_b \delta}} \widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b \delta)}} \Phi'^{\frac{\delta(\varepsilon_b - \eta)}{\varepsilon_b (1+\varepsilon_b \delta)}}_m \left(\frac{\Phi'}{L^S} \right)^{\frac{\delta}{1+\varepsilon_b \delta}}, \tag{C.6}$$

where the employer components in the counterfactual: $\left(\frac{\beta_b \lambda_i' \overline{Z}_i'}{(T_i \Gamma_b^{\eta})^{\delta}} \right)^{\frac{1}{1+\varepsilon_b \delta}}$, also include are enough to compute the labor supply shares, as shown in Propositions 7 and 2. Following the same steps as in the baseline, the sector level system of equations in the counterfactual is analogous to (12) but with relative variables. Solving for relative sector prices we can compute the sector labor supply $L_i^{S\prime}$. Proposition 4 also apply in the "hat" economy, and if $\psi_i = 1$, also Proposition 8. Section S1.5 of the Supplemental Material contains more details.

C.5 Non targeted moments

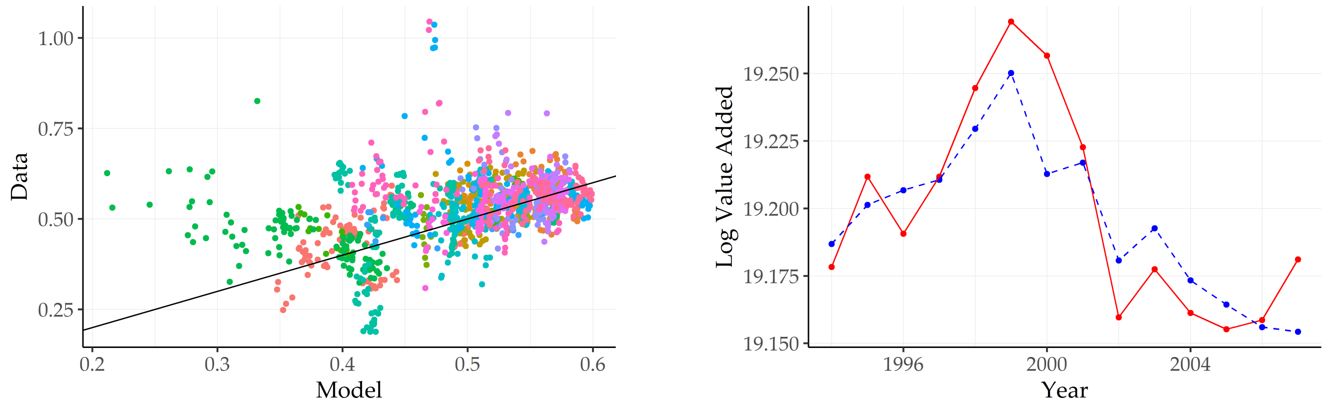
Panel (a) of Figure C.2 presents 3-digit industry labor shares per year. The horizontal axis shows model generated moments, while the vertical axis has the corresponding observed moment in the data. If the fit was perfect, each dot would be on the 45 degree line. Each color represents a 2-digit industry. We see that most of the dots are aligned around the 45 degree line.

Panel (b) shows the model matches the evolution of aggregate value added. Since there is a strong link between production and wage bill and the model matches wages and labor allocations exactly, it also has a good fit of the value added.

D Counterfactuals

We present additional results from counterfactuals. The details of the counterfactual with employer exit are in the Supplemental Material.

Figure C.2: Model Fit Non Targeted Moments



(a) Sub-industry labor share

(b) Y. Model (dashed blue), data (red).

D.1 Increasing union bargaining powers: Output

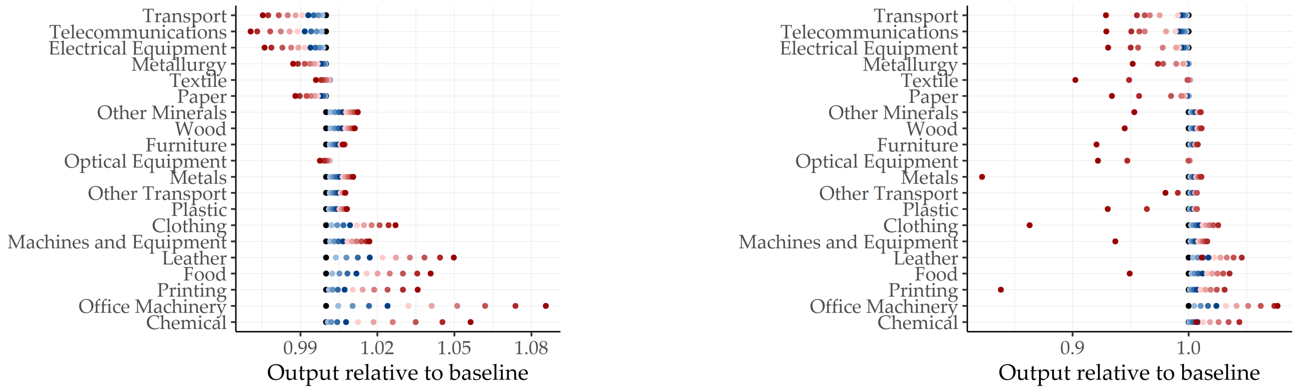
Figure D.1 resents output effects of increasing union bargaining powers by increasing κ for two union objectives. Figure D.1a takes the baseline union objective of utilitarian unions. It shows that, in line with the results in Figure 5b, the sectors at the bottom that features the highest increases in productivity also experience, as a group, the highest output gains. On the contrary, sectors at the top of the figure reduce their employment relative to their baseline as employment shifts towards the sectors with highest productivity and wage increases.

Figure D.1b replicates the analysis when the unions have misaligned objectives on insiders. These objectives lead to labor rationing for high enough union bargaining powers. Contrary to Figure D.1a, output losses are more significant as high enough φ_b generate unemployment.

D.2 The importance of labor mobility

We check three additional cases to locate the output changes in an environment with mobility costs. These cases differ in their mobility restrictions, where we allow mobility to happen only within sector, sector-occupation and local labor market. Table D.1 compares the free mobility case with the restricted mobility cases for different counterfactuals. Comparing the output changes in column 3 across the different scenarios, we find that restricting mobility reduces the output gains from removing the labor wedges. In the *Oligopsony* counterfactual, when labor is constrained to remain in the local labor market, output does not decrease as much as in the

Figure D.1: Increasing Union Bargaining Powers: Output effects of κ



(a) Baseline objectives

(b) Misaligned objectives

Notes: Sector output relative to the baseline for different κ. The union bargaining power is $\varphi_b^{1-\kappa}$, where blue (red) denotes low (high) values of κ. Panel (a): utilitarian union (Section 2.7). Panel (b): misaligned union objectives (Section 5.4).

free mobility case. However, in the other two cases, *Fixed sector* and *Fixed sector-occupation*, the output losses are greater. In the oligopsonistic competition counterfactual, fixing employment across sectors but allowing for geographical mobility exacerbates the output losses. Restricting employment to move only within a local labor market would contain output losses as productivity losses are reduced by more than 60%.

Fixing employment at the sector-occupation level accounts for 84% of the gains of the free mobility case without labor wedges. While restricting workers to stay in their particular local labor market output gains are 0.48%, which constitute only 29% of the gains under free mobility without wedges. Comparing the free mobility counterfactuals to the ones with restricted labor mobility we see that the key margin of adjustment is geographical mobility.

These results underscore the importance of free mobility of labor to counteract the output losses from the misallocation coming from heterogeneous wedges. The left panel of Figure D.2 shows the percentage change of manufacturing employment in the free mobility case in the oligopsonistic competition counterfactual. Each block is the aggregation of local labor markets to the commuting zone. In the absence of unions, manufacturing employment in the rural areas would be reduced. The counterfactual reveals that there are a handful of rural productive establishments in concentrated local labor markets. In the baseline these have lower wage markdowns and lower employment that are further dampened without unions. Moving to

Table D.1: Counterfactuals: Limited Mobility

		Labor share (%)	ΔY	Contribution Δ Y (%) GE	Prod	Labor
Oligopsony	Free mobility	36.48	-0.67	-12.21	169.02	-56.81
	Fixed sector	36.48	-1.11	-1.62	101.62	0
	Fixed sector-occ	36.47	-1.1	-2.38	102.38	0
	Fixed local market	35.96	-0.44	-2.7	102.7	0
No wedges	Free mobility	73.33	1.65	8.32	85.55	6.13
	Fixed sector	73.33	1.39	-1.29	101.29	0
	Fixed sector-occ	73.33	1.39	-1.86	101.86	0
	Fixed local market	73.33	0.48	-1.33	101.33	0

Notes: $\Delta x \equiv (x' - x)/x$. Results in percentages. ΔY: output gains. Last columns decompose ΔY as (S1.9). *Free mobility*: without mobility restrictions, *Fixed sector*: mobility only within sector, *Fixed sector-occ*: fixed employment at sector-occupation (location and 3-digit industry mobility), and *Fixed local market*: mobility only within m.

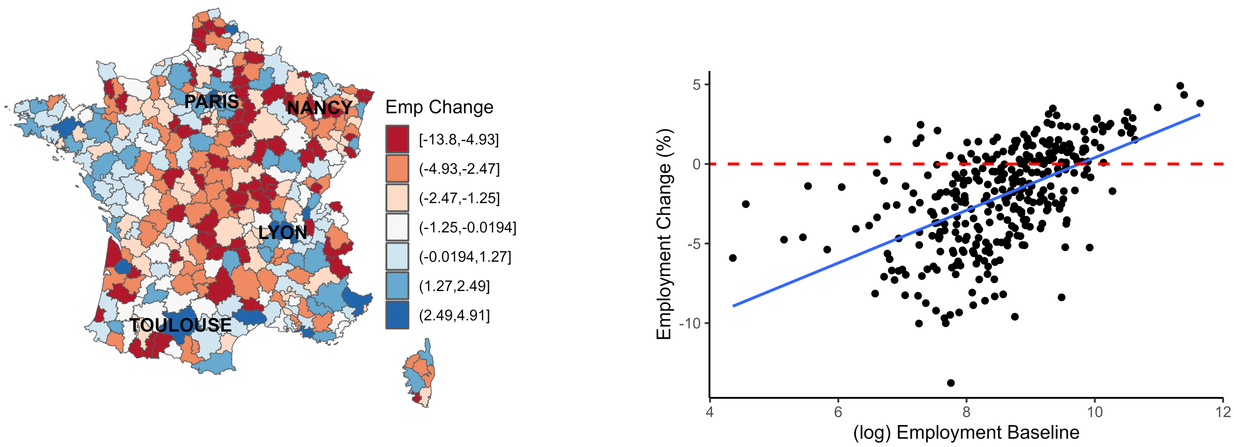
the counterfactual, those are the ones with the biggest relative wage and employment losses. The right panel of Figure D.2 shows a positive relationship between the logarithm of baseline employment at the commuting zone and employment gains in the oligopsonistic competition counterfactual without unions.[10] Rural areas or commuting zones with low employment levels in the baseline are the ones that benefit the most from the presence of unions.

Columns 3 to 5 of Table D.1 show the decomposition of relative changes of output from equation (S1.9) in the Supplemental Material. The main source of output changes come from productivity because sector productivity is an employment weighted sum of establishment-occupation productivities (which are unchanged). The source of aggregate productivity and output losses without unions is therefore the reallocation of workers towards less productive establishments. Sectoral mobility exacerbates the negative productivity effects of removing unions under free mobility. When restricting mobility by keeping employment constant at the local labor market level, the misallocation effects are curbed and output changes to -0.44%.

The Supplemental Material shows that in *Oligopsony*, largest productivity losses happen outside urban areas. As a result, the largest wages and employment losses happen in commuting zones that do not include big cities.

[10] The Supplemental Material shows the analogous for the counterfactual without labor wedges.

Figure D.2: Employment Change (%) in the Counterfactual: Oligopsonistic Competition



Notes: Left: commuting zone employment changes in *Oligopsony* from the baseline. Right: employment change versus the log of employment in the baseline, the blue line is fitted from an OLS.

Table D.2: Wage Gap

	Rural Wage	Urban Wage	Gap (%)
Baseline	33.30	44.53	33.71
Counterfactual: Oligopsony	22.99	34.02	47.97
Counterfactual: No wedges	49.98	58.70	17.45

Notes: Wages in thousands of constant 2015 euros. *Urban*: 10 biggest commuting zones: Paris, Marseille, Lyon, Toulouse, Nantes, and the Paris surrounding areas of Boulogne-Billancourt, Creteil, Montreuil, Saint-Denis and Argenteuil. The rest are considered as *Rural*. Wages are employment weighted averages per location in 2007.

D.3 The effect of labor market power on urban-rural differences

Figure D.2 suggests an important labor reallocation from rural areas to cities in the counterfactual without unions. Table D.2 presents the impact of employer and union labor market power on the urban-rural wage gap. Both urban and rural areas experience important wage changes in the counterfactuals. Under oligopsonistic competition, the urban-rural wage gap amplifies from 34% to 48% and is cut up to 17% without labor wedges. This reveals that labor market distortions account for half of the urban-rural wage gap.

Table D.3: Counterfactual: Endogenous Participation

	Δ L(%)	ΔY(%)	Contribution Δ Y (%) GE	Prod	Labor
Fixed L	0	-0.67	-12.21	169.02	-56.81
Endogenous Participation					
Oligopsony $\lambda(\mu, 0) = \mu_i$	-0.93	-1.57	-5.19	71.82	33.36
No wedges $\lambda(1, 0) = 1$	0.86	2.49	5.52	56.79	37.68

Notes: Results in percentages. ΔL: employment gains; ΔY: output gains. Last columns decompose ΔY as (S1.9). *Fixed L*: *Oligopsony* in Table 2. Other counterfactuals allow for endogenous labor force participation.

D.4 Extensions

The main counterfactual assumed that the total labor supply was fixed and there were no agglomeration externalities. Here we present results from counterfactuals that relax these assumptions allowing for an endogenous labor participation decision, and for agglomeration forces. All the details are left for the Supplemental Material.

D.4.1 Endogenous labor force participation

In this extension we assume workers can decide between working and staying at home where, they earn wages related to home production so participation is now an endogenous choice.

Table D.3 shows the results. Introducing an endogenous labor force participation margin induces higher output losses, -1.57%, than in the baseline (*Fixed L*) as the total labor supply decreases by 0.93%. In contrast to the output decomposition in Table D.1, 33% of the losses come from employment. This extensive margin of adjustment in the total labor supply slightly amplifies the original differences in output gains across counterfactuals. Output gains without labor wedges are 2.49% because total labor force participation increases by 0.86%. Despite featuring high wage changes, the differences in total labor supply are minor because we assume that workers have idiosyncratic shocks to stay out of the labor force and their mobility is governed by the across-market elasticity of substitution. This elasticity is low as $\widehat{\eta} = 0.332$.

Table D.4: Counterfactual: Agglomeration

	ΔY (%)	Contribution Δ Y (%)		
		GE	Prod	Labor
No Agglomeration	-0.67	-12.21	169.02	-56.81
Agglomeration				
$\gamma = 0.05$	-1.86	61.2	56.89	-18.09
$\gamma = 0.1$	-3.19	78.85	30.08	-8.93
$\gamma = 0.2$	-6.24	92.2	10.43	-2.63
$\gamma = 0.25$	-7.93	96.06	5.15	-1.2
$\gamma = 0.3$	-9.69	99.38	0.84	-0.22

Notes: Results in percentages. ΔY: output gains. Last columns decompose ΔY as (S1.9). *No Agglomeration*: *Oligopsony* in Table 2. Other counterfactuals allow for agglomeration within m that depends on γ.

D.4.2 Agglomeration

We extend the model to include agglomeration forces at the local labor market level. We assume that the agglomeration effect is a local labor market externality with elasticity $\gamma(1-\alpha_b)$.

Table D.4 summarizes the counterfactuals removing unions for different values of γ under oligopsonistic competition, free mobility and fixed total employment. As γ becomes higher, the more important are the agglomeration forces and the more exacerbated are the output losses.

D.5 Sector wage floors

Table D.5 present the results of bargaining of wage floors at the sector level. The columns *Objective*, *Productivity*, *Wage Bill* and *Profit* present sector aggregates in the counterfactuals with sector wage floors relative to a counterfactual without minimum wages where employers compete oligopsonistically. We consider counterfactuals where we set the sector wage floors to the 1st, 5th and 10th percentiles of the observed wages. We compute the Nash objective $\mathbf{G}\left(\underline{w}_b\right)^{\varphi_b}\mathbf{\Pi}\left(\underline{w}_b\right)^{1-\varphi_b}$ where we assume the aggregate union's objective $\mathbf{G}\left(\underline{w}_b\right)$ is the sector wage bill and the employer association cares about sector profits $\mathbf{\Pi}\left(\underline{w}_b\right)$. Evaluating the aggregate variables relative to the oligopsony counterfactual already reveals that the wage floor maximizing the bargaining objective is either zero or very low as the all the sectors reach lower levels of the Nash product than in oligopsony even for the low wage floor *P1*.[11] Even if most sectors find wage bill

[11] Section S9.6 of the Supplemental Material shows the distribution of employers per region with *P1* wage floors.

Table D.5: Counterfactuals: Sector Wage Floors

	Objective			Productivity			Wage Bill			Profit		
Sector	P1	P5	P10	P1	P5	P10	P1	P5	P10	P1	P5	P10
Food	-0.4	-3.1	-8.2	-0.2	-0.7	-2.5	0.1	0.6	1.5	-0.6	-4.1	-10.9
Textile	-2.2	-18.4	-25.8	-1.7	-13.8	-17.6	-0.2	2.6	7.9	-4.3	-35.9	-50.0
Clothing	-0.6	-4.2	-10.3	-0.3	-1.5	-3.5	0.1	0.3	0.3	-0.8	-6.2	-14.6
Leather	-4.2	-18.2	-28.3	-2.1	-8.5	-11.9	0.1	0.7	1.7	-5.8	-24.3	-37.1
Wood	-1.5	-20.3	-33.0	-1.0	-14.3	-21.1	0.0	2.9	8.4	-2.6	-34.0	-53.0
Paper	-2.2	-10.9	-18.4	-0.8	-5.0	-8.3	-0.4	0.2	1.1	-4.0	-21.2	-34.8
Printing	-0.2	-1.5	-3.8	-0.1	0.0	-0.3	0.1	0.2	0.3	-0.3	-1.9	-4.7
Chemical	-0.1	-1.1	-2.6	0.1	0.0	-0.3	0.0	-0.1	-0.1	-0.1	-1.2	-2.9
Plastic	-0.9	-11.6	-21.0	-0.2	-4.3	-7.6	0.0	0.7	2.1	-1.4	-17.5	-31.1
Other Minerals	-2.1	-16.4	-24.6	-1.5	-11.3	-15.6	0.0	1.8	5.1	-3.6	-28.1	-41.5
Metallurgy	-9.4	-17.9	-21.2	-6.5	-10.3	-10.9	-0.7	-0.3	0.8	-20.0	-36.8	-43.4
Metals	-1.0	-11.6	-19.9	-0.7	-8.7	-14.0	0.1	1.8	5.2	-1.9	-20.2	-34.2
Machines and Equipment	-0.5	-5.0	-9.9	0.0	-1.6	-3.3	-0.1	-0.3	-0.1	-0.7	-6.9	-13.9
Office Machinery	-0.1	-9.2	-21.8	0.0	-4.1	-9.7	0.0	-0.3	-0.6	-0.1	-11.0	-25.8
Electrical Equipment	-4.0	-10.0	-13.4	-2.1	-4.7	-5.9	-1.2	-2.1	-1.9	-8.5	-21.5	-29.6
Telecommunications	-1.6	-4.8	-7.0	-0.4	-1.3	-1.9	-0.8	-2.0	-2.5	-3.1	-10.0	-14.9
Optical Equipment	-0.7	-5.2	-8.7	-0.2	-2.1	-3.5	-0.1	-0.6	-0.4	-1.1	-8.4	-14.3
Transport	-3.1	-10.1	-14.7	-2.2	-5.6	-7.1	-0.6	-1.6	-1.7	-8.3	-25.7	-36.9
Other Transport	-0.5	-3.4	-7.0	0.0	-0.5	-1.4	-0.1	-0.8	-1.3	-0.7	-5.1	-10.5
Furniture	-0.5	-7.9	-18.1	-0.3	-4.4	-9.8	0.1	0.4	1.9	-0.9	-13.6	-30.2

Notes: Counterfactuals with sector wage floors equal to the 1st (*P1*), 5th (*P5*) and 10th (*P10*) percentiles of the observed wages. Sector *Objective*: Nash product between wage bill and profits; *Productivity*, *Wage Bill* and *Profit*.

increases from the introduction of the evaluated wage floors, the sharp reduction of aggregate profits with minimum wages, induces lower Nash products than in oligopsony.

D.6 Unions

Table D.6 presents union density and coverage statistics for several countries.[12] France has the highest coverage.

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Table D.6: Union Density and Collective Bargaining Coverage

Country	Union Density	Coverage	Country	Union Density	Coverage
Western Europe			**Southern Europe**		
Austria	27.7	98.0	Italy	36.4	80.0
France	9.0	98.5	Spain	16.8	80.2
Germany	17.7	57.8	**Americas**		
Netherlands	18.1	85.9	Canada	29.3	30.4
Switzerland	16.1	49.2	Chile	15.3	19.3
Northern Europe			United States	10.7	12.3
Finland	67.6	89.3	**Asia & Oceania**		
Ireland	26.3	33.5	Australia	15.1	59.9
Norway	49.7	67.0	Japan	17.5	16.9
United Kingdom	25.0	27.5	Korea	10.0	11.9

Notes: Year 2014. Variables in percents. *Union Density*: unionization rate (unionized workers relative to total employment), *Coverage*: collective agreement coverage (ratio of employees covered by collective agreements divided by all wage earners with the right to bargain). OECD data from administrative data except for Australia, Ireland and the United States which are based on survey data. Regions according to the U.N. M49 area codes.

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