Supplemental Material - Not For Publication

Union and Firm Labor Market Power

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This Supplemental Material is organized as follows. Section S1 presents additional derivations, which includes the establishment labor supply, characterizations of labor wedges and rationing shares, and the model aggregation. Section S2 contains details of the extensions. Section S3 presents additional proofs and results on the model aggregation. Section S4 contains details of sector wage floors. Section S5 presents the derivations with a representative household. Section S6 contains the derivations for the social welfare measure and its decomposition. Section S7 contains a detailed derivation of the bias of the reduced-form estimate of the labor supply elasticity. Section S8 contains additional estimation results. Section S9 contains additional results from counterfactuals. Section S10 presents details on sample selection and variable construction. Section S11 presents summary statistics.

S1 Additional derivations

S1.1 Employer labor supply

The indirect utility of a worker ι who is employed in i within market m is:

$$\mathcal{U}_i(\iota) = \psi_i w_i z_i(\iota) u_m(\iota) = \overline{w}_i z_i(\iota) u_m(\iota),$$

where $z_i(\iota)$ and $u_m(\iota)$ are idiosyncratic taste shocks with cumulative distribution functions $\mathcal{F}_{z,i}$ and $\mathcal{F}_u$. The first shock, $z_i(\iota)$ is employer-specific, while the second applies to all employers within local labor market m. They are both distributed Fréchet, so $\mathcal{F}_{z,i}(z) = e^{-T_i z^{-\varepsilon_b}}$, $T_i > 0$, $\varepsilon_b > 1$ and $\mathcal{F}_u(u) = e^{-u^{-\eta}}$, $\eta > 0$.

Workers first observe shocks u for all local labor markets. After choosing their labor market, workers then observe the employer specific shocks. Therefore, there is a two stage decision:

first, the worker chooses the local labor market that maximizes her expected utility, and subsequently she chooses the establishment that maximizes her utility conditional on the chosen sub-market.

The unconditional probability of a worker going to establishment i in market m is:

$$s_i = \mathcal{P}\left(\overline{w}_i z_i \geq \max_{i' \neq i} \overline{w}_{i'} z_{i'} \middle| \{i, i'\} \in \mathcal{I}_m\right) \mathcal{P}\left(\mathbb{E}(\max_{j \in \mathcal{I}_m} \overline{w}_j z_j) u_m \geq \max_{m' \neq m} \mathbb{E}(\max_{j \in \mathcal{I}_{m'}} \overline{w}_j z_j) u_{m'}\right).$$

We first solve for the left term. Let's define the following CDF:

$$G_i(v) = \mathcal{P}\left(\overline{w}_i z_i < v\right) = \mathcal{P}\left(z_i < v/\overline{w}_i\right) = e^{-T_i \overline{w}_i^{\varepsilon_b} v^{-\varepsilon_b}}.$$

To ease notation, define the *conditional* utility $v_i = \overline{w}_i z_i$. The first term becomes:

$$\mathcal{P}\left(v_i \geq \max_{i' \neq i} v_{i'} \middle| \{i, i'\} \in \mathcal{I}_m\right).$$

Fix $v_i = v$. Then we have, for all $i' \in m$:

$$\mathcal{P}\left(v \geq \max_{i' \neq i} v_{i'} \middle| \{i, i'\} \in \mathcal{I}_m\right) = \bigcap_{i' \neq i} \mathcal{P}\left(v_{i'} < v\right) = \prod_{i' \neq i} G_{i'}(v) = e^{-\Phi_m^{-i} v^{-\varepsilon_b}} = G_m^{-i}(v),$$

where $\Phi_m^{-i} \equiv \sum_{i' \neq i} T_{i'} \overline{w}_{i'}^{\varepsilon_b}$. Similarly, the probability of having at most conditional utility v is: $G_m(v) = \mathcal{P}\left(v \geq \max_{i'} v_{i'}\right) = e^{-\Phi_m v^{-\varepsilon_b}}$, where $\Phi_m \equiv \sum_{i'} T_{i'} \overline{w}_{i'}^{\varepsilon_b}$.

Integrating $G_m^{-i}(v)$ over all possible values of v:

$$\begin{aligned}\mathcal{P}\left(v_i \geq \max_{i' \neq i} v_{i'} \middle| \{i, i'\} \in \mathcal{I}_m\right) &= \int_0^\infty e^{-\Phi_m^{-i} v^{-\varepsilon_b}} dG_i(v) = \int_0^\infty \varepsilon_b T_i \overline{w}_i^{\varepsilon_b} v^{-\varepsilon_b - 1} e^{-\Phi_m v^{-\varepsilon_b}} dv \\ &= \frac{T_i \overline{w}_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b - 1} e^{-\Phi_m v^{-\varepsilon_b}} dv = \frac{T_i \overline{w}_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty dG_m(v) = \frac{T_i \overline{w}_i^{\varepsilon_b}}{\Phi_m}.\end{aligned}$$

Now we need to get $\mathcal{P}\left(\mathbb{E}(\max_{j \in \mathcal{I}_m} \overline{w}_j z_j) u_m \geq \max_{m' \neq m} \mathbb{E}(\max_{j \in \mathcal{I}_{m'}} \overline{w}_j z_j) u_{m'}\right)$. First, we have that the expected utility of working in market m is:

$$\mathbb{E}(\max_{i \in \mathcal{I}_m} \overline{w}_i z_i) = \int_0^\infty v_i dG_m(v) = \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b} e^{-\Phi_m v^{-\varepsilon_b}} dv.$$

We define a new variable: $x = \Phi_m v^{-\varepsilon_b}, dx = -\varepsilon_b \Phi_m v^{-(\varepsilon_b + 1)} dv$. Now we can change variable in the previous integral and obtain:

$$\int_0^\infty x^{-1/\varepsilon_b}\Phi_m^{1/\varepsilon_b}e^{-x}dx = \widetilde{\Gamma}\left(\frac{\varepsilon_b - 1}{\varepsilon_b}\right)\Phi_m^{1/\varepsilon_b},$$

where $\widetilde{\Gamma}(\cdot)$ is the Gamma function. Defining $\Gamma_b \equiv \widetilde{\Gamma}\left(\frac{\varepsilon_b-1}{\varepsilon_b}\right)$, we can rewrite:

$$\mathcal{P}\left(\mathbb{E}(\max_{j\in\mathcal{I}_m} \overline{w}_j z_j)u_m \geq \max_{m'\neq m} \mathbb{E}(\max_{j\in\mathcal{I}_{m'}} \overline{w}_j z_j)u_{m'}\right) = \mathcal{P}\left(\Phi_m^{1/\varepsilon_b}\Gamma_b u_m \geq \max_{m'\neq m} \Phi_{m'}^{1/\varepsilon_{b'}}\Gamma_{b'}u_{m'}\right).$$

Following similar arguments as above:

$$\mathcal{P}\left(\Phi_m^{1/\varepsilon_b}\Gamma_b u_m \geq \max_{m'\neq m} \Phi_{m'}^{1/\varepsilon_{b'}}\Gamma_{b'}u_{m'}\right) = \frac{\Phi_m^{\eta/\varepsilon_b}\Gamma_b^\eta}{\Phi},$$

where $\Phi \equiv \sum_{b'\in\mathcal{B}}\sum_{m'\in\mathcal{M}_{b'}} \Phi_{m'}^{\eta/\varepsilon_{b'}}\Gamma_{b'}^\eta$. Finally, combining the two probabilities we obtain:

$$s_i = \frac{T_i\overline{w}_i^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b}\Gamma_b^\eta}{\Phi}.$$

By integrating s_i over the measure of workers L^S, i's labor supply is: $L_i^S = s_i \times L^S$.

S1.2 Labor wedge

Here we do the derivations from the bargaining problem (2) that gives rise to equations (3) and (4) on the paper.

First, consider the case where the labor demand constraint is active. Then, the wage trivially is equal to $w_i = F_i'(L_i)$.

In the second case, the labor demand constraint is active and $\psi_i = 1$. Then, the bargaining problem is:

$$\max_{w_i} (G_i(w_i))^\varphi \left(F_i(\ell_i^S(w_i)) - w_i\ell_i^S(w_i)\right)^{1-\varphi},$$

where we already incorporated the constraint of $\psi_i = 1 \iff \ell_i^D(w_i) = \ell_i^S(w_i)$. The first order conditions are:

$$\varphi \left(\frac{G_i}{F_i(L_i) - w_i L_i} \right)^{-1} \frac{\partial G_i}{\partial w_i} + (1-\varphi) \left(F'(L_i) \frac{\partial \ell_i^S}{\partial w_i} - L_i - w_i \frac{\partial \ell_i^S}{\partial w_i} \right) = 0.$$

Using the elasticity of the union objective to wages $\xi(G_i, w_i) \equiv \frac{\partial G_i}{\partial w_i} \frac{w_i}{G_i}$ and the elasticity of labor supply e_i, the above simplifies to:

$$\varphi\, \xi(G_i, w_i)\, \frac{F_i(L_i) - w_i L_i}{w_i L_i} + (1-\varphi) F'(L_i) e_i \frac{1}{w} = (1-\varphi)(1+e_i).$$

Rearranging and using $\nu_i \equiv F'(L_i) \frac{L_i}{F(L_i)}$:

$$\varphi \frac{\xi(G_i, w_i)}{1+e_i} \frac{1}{\nu_i} F'(L_i) + (1-\varphi) F'(L_i) \frac{e_i}{1+e_i} = w_i \left[\varphi \frac{\xi(G_i, w_i)}{1+e_i} + (1-\varphi) \right].$$

Factoring out the MRPL, $F'(L_i)$, and rearranging we have that the bargained wage is:

$$\implies w_i = \left[\frac{\varphi\, \xi(G_i, w_i) \frac{1}{\nu_i} + (1-\varphi) \frac{e_i}{1+e_i}}{\varphi\, \xi(G_i, w_i) + (1-\varphi)(1+e_i)} \right] F'(L_i), \tag{S1.1}$$

We can rewrite the wedge in S1.1 as:

$$\widetilde{\lambda}_i = \omega_i \frac{1}{\nu_i} + (1-\omega_i) \frac{e_i}{1+e_i},$$

where

$$\omega_i = \frac{\varphi \xi(G_i, w_i)}{\varphi \xi(G_i, w_i) + (1-\varphi)(1+e_i)}.$$

This is exactly the expression in (3) with $\mu_i = \frac{e_i}{1+e_i}$ and $\widetilde{\lambda}_i = \lambda_i$.

Combining the two possible cases corresponding to which constraint is active yields equation (3).

S1.3 Rationing

In this section we provide the key derivations of the model with rationing that are not presented in the main text. We first derive some useful elasticities when the allocation is determined by the labor demand. To ease up on notation we abstract from sector-specific parameters. So, $\varphi_b = \varphi$, etcetera.

Expected wage and rationing along the labor demand. After choosing capital, the revenue of employer i is:

$$F_i(L_i) = \widetilde{Z}_i L_i^{1-\delta},$$

where $\widetilde{Z}_i = (1-\alpha_b) P_b^{\frac{1}{1-\alpha_b}} A_i$. Maximizing profits for a given bargained wage w_i we get the following first order condition:

$$w_i = (1-\delta)\widetilde{Z}_i L_i^{-\delta}.$$

Inverting this equation we get the labor demand:

$$\ell_i^D(w_i) = \left((1-\delta)\widetilde{Z}_i\right)^{1/\delta} w_i^{-1/\delta}.$$

Using the definition of $\psi_i = \ell_i^D(w_i)/\ell_i^S(\overline{w}_i)$ we get:

$$\psi_i = \frac{\left((1-\delta)\widetilde{Z}_i\right)^{1/\delta} w_i^{-1/\delta}}{\ell_i^S(\overline{w}_i)}.$$

This implies that, along the labor demand ($L_i = \ell_i^D(w_i)$), the expected wage is:

$$\overline{w}_i = \frac{\left((1-\delta)\widetilde{Z}_i\right)^{1/\delta} w_i^{-(1-\delta)/\delta}}{\ell_i^S(\overline{w}_i)}.$$

Elasticity of expected wage wrt to wage. First we take the derivative of $\overline{w}_i$ with respect to w_i:

$$\frac{d\overline{w}_i}{dw_i} = \frac{\partial \overline{w}_i}{\partial w_i} + \frac{\partial \overline{w}_i}{\partial \ell_i^S}\frac{d\ell_i^S}{dw_i}.$$

Developing $\frac{d\ell_i^S}{dw_i}$, this is equal to:

$$\begin{aligned}\frac{d\overline{w}_i}{dw_i} &= \frac{\partial \overline{w}_i}{\partial w_i} + \frac{\partial \overline{w}_i}{\partial \ell_i^S}\frac{\partial \ell_i^S}{\partial \overline{w}_i}\frac{d\overline{w}_i}{dw_i} \\ &= \frac{\partial \overline{w}_i}{\partial w_i}\left(1 - \frac{\partial \overline{w}_i}{\partial \ell_i^S}\frac{\partial \ell_i^S(\overline{w}_i)}{\partial \overline{w}_i}\right)^{-1}.\end{aligned}$$

Using $\frac{\partial \overline{w}_i}{\partial \ell_i^S} = -\frac{\overline{w}_i}{\ell_i^S}$ and substituting above we get:

$$\frac{d\overline{w}_i}{dw_i} = \frac{\partial \overline{w}_i}{\partial w_i}\left(1 + \frac{\partial \ell_i^S(\overline{w}_i)}{\partial \overline{w}_i}\frac{\overline{w}_i}{\ell_i^S}\right)^{-1} = \frac{\partial \overline{w}_i}{\partial w_i}\left(1+e_i\right)^{-1},$$

where e_i is the labor supply elasticity. Now, using $\frac{\partial \overline{w}_i}{\partial w_i} = -\frac{1-\delta}{\delta}\frac{\overline{w}_i}{w_i}$, we get:

$$\frac{d\overline{w}_i}{dw_i} = -\frac{1-\delta}{\delta}\frac{\overline{w}_i}{w_i}\left(1+e_i\right)^{-1}. \tag{S1.2}$$

Therefore, the elasticity is equal to:

$$\frac{d\overline{w}_i}{dw_i}\frac{w_i}{\overline{w}_i} = -\frac{1-\delta}{\delta}\frac{1}{1+e_i} = -\frac{1}{\delta}\left(\frac{1-\delta}{1+e_i}\right). \tag{S1.3}$$

Elasticity rationing share wrt wage. We follow similar steps for this elasticity. We start with the derivative of rationing share with respect to the wage:

$$\frac{d\psi_i}{dw_i} = \frac{\partial \psi_i}{\partial w_i} + \frac{\partial \psi_i}{\partial \ell_i^S}\frac{d\ell_i^S}{dw_i}.$$

Developing $\frac{d\ell_i^S}{dw_i}$, this is equal to:

$$\frac{d\psi_i}{dw_i} = \frac{\partial\psi_i}{\partial w_i} + \frac{\partial\psi_i}{\partial\ell_i^S}\frac{\partial\ell_i^S}{\partial\overline{w}_i}\frac{d\overline{w}_i}{dw_i}.$$

Substituting $\frac{d\overline{w}_i}{dw_i}$ above using (S1.2):

$$\frac{d\psi_i}{dw_i} = \frac{\partial\psi_i}{\partial w_i} + \frac{\partial\psi_i}{\partial\ell_i^S}\frac{\partial\ell_i^S}{\partial\overline{w}_i}\left(-\frac{1-\delta}{\delta}\frac{1}{1+e_i}\frac{\overline{w}_i}{w_i}\right).$$

Taking $-\frac{\overline{w}_i}{w_i}$ out of the parenthesis and using $\frac{\partial\psi_i}{\partial\ell_i^S} = -\frac{\psi_i}{\ell_i^S}$ we get:

$$\frac{d\psi_i}{dw_i} = \frac{\partial\psi_i}{\partial w_i} + \frac{\psi_i}{\ell_i^S}\frac{\partial\ell_i^S}{\partial\overline{w}_i}\frac{\overline{w}_i}{w_i}\left(\frac{1-\delta}{\delta}\frac{1}{1+e_i}\right).$$

Rearranging, and using $e_i = \frac{\partial\ell_i^S}{\partial\overline{w}_i}\frac{\overline{w}_i}{\ell_i^S}$:

$$\frac{d\psi_i}{dw_i} = \frac{\partial\psi_i}{\partial w_i} + \frac{\psi_i}{w_i}\left(\frac{1-\delta}{\delta}\frac{e_i}{1+e_i}\right).$$

We also have that $\frac{\partial\psi_i}{\partial w_i} = -\frac{1}{\delta}\frac{\psi}{w_i}$. Substituting above we get:

$$\frac{d\psi_i}{dw_i} = -\frac{1}{\delta}\frac{\psi_i}{w_i} + \frac{\psi_i}{w_i}\left(\frac{1-\delta}{\delta}\frac{e_i}{1+e_i}\right).$$

Therefore, the elasticity is equal to:

$$\frac{d\psi_i}{dw_i}\frac{w_i}{\psi_i} = -\frac{1}{\delta} + \left(\frac{1-\delta}{\delta}\frac{e_i}{1+e_i}\right) = -\frac{1}{\delta}\left(\frac{1+\delta e_i}{1+e_i}\right). \tag{S1.4}$$

Maximization Nash product. If the labor demand is binding, ψ_i is a function of w_i. Then the union's objective function is only a function of w_i. Let $\Pi_i(w_i)$ be the profit for employer i after substituting the labor demand. Then, the Nash product is:

$$G_i(w_i)^{\varphi}\Pi_i(w_i)^{1-\varphi},$$

By maximizing the Nash product, we get the following optimality condition:

$$\varphi \frac{dG_i}{dw_i} \frac{w_i}{G_i} = -(1-\varphi) \frac{d\Pi_i}{dw_i} \frac{w_i}{\Pi_i}.$$

The profit of employer i is:

$$\Pi_i(w_i) = \delta \widetilde{Z}_i L_i^{1-\delta}.$$

Then the elasticity of profit with respect to the wage is:

$$\frac{d\Pi_i}{dw_i} \frac{w_i}{\Pi_i} = \underbrace{\frac{d\Pi_i}{dL_i} \frac{L_i}{\Pi_i}}_{1-\delta} \times \underbrace{\frac{dL_i}{dw_i} \frac{w_i}{L_i}}_{-1/\delta} = -\frac{1-\delta}{\delta},$$

where we used that employment is along the labor demand ($L_i = \ell_i^D(w_i)$). The optimality condition becomes:

$$\varphi \frac{dG_i}{dw_i} \frac{w_i}{G_i} = (1-\varphi) \frac{1-\delta}{\delta}. \tag{S1.5}$$

We only need to find the union's objective function elasticity with respect to the wage.

Elasticity of union's objective function. The 'misaligned' objective function is given by:

$$G_i(w_i) = \left[\frac{(\Phi_m)^{1/\varepsilon}}{\psi_i} - (\Phi_{m,-i})^{1/\varepsilon} \right] \psi_i \ell_i^S(\overline{w}_i),$$

where

$$\Phi_m = \sum_{j \in \mathcal{I}_m} T_j \overline{w}_j^{\varepsilon}, \quad \text{and} \quad \Phi_{m,-i} = \sum_{j \in \mathcal{I}_m \setminus \{i\}} T_j \overline{w}_j^{\varepsilon} = \Phi_m - T_i \overline{w}_i^{\varepsilon}.$$

Both ψ_i and $\overline{w}_i$ are functions of w_i as the labor demand is binding.

We can separate the elasticity to ease up the derivations. More specifically:

$$\frac{d\log G_i}{d\log w_i} = \frac{d\log \overbrace{\left((\Phi_m)^{1/\varepsilon} - \psi_i \left(\Phi_{m,-i}\right)^{1/\varepsilon} \right)}^{G_1}}{d\log w_i} + \frac{d\log \ell_i^S(\overline{w}_i)}{d\log w_i}.$$

Let us start by taking the derivative of G_1:

$$\frac{dG_1}{dw_i} = \frac{1}{\varepsilon}\left(\Phi_m\right)^{1/\varepsilon - 1} \varepsilon T_i \overline{w}_i^{\varepsilon-1} \frac{d\overline{w}_i}{dw_i} - \left(\Phi_{m,-i}\right)^{1/\varepsilon} \frac{d\psi_i}{dw_i}.$$

Recall that $s_{i|m} = \frac{T_i \overline{w}_i^{\varepsilon}}{\Phi_m}$. Then, we have that:

$$\frac{dG_1}{dw_i} = \left(\Phi_m\right)^{1/\varepsilon} \frac{s_{i|m}}{\overline{w}_i} \frac{d\overline{w}_i}{dw_i} - \left(\Phi_{m,-i}\right)^{1/\varepsilon} \frac{d\psi_i}{dw_i}.$$

Factorizing Φ_m and $1/w_i$ and multiplying and dividing by ψ_i the second term we obtain:

$$\frac{dG_1}{dw_i} = \frac{\Phi_m^{1/\varepsilon}}{w_i} \left(s_{i|m} \frac{d\overline{w}_i}{dw_i} \frac{w_i}{\overline{w}_i} - \psi_i (1 - s_{i|m})^{1/\varepsilon} \frac{d\psi_i}{dw_i} \frac{w_i}{\psi_i} \right),$$

where we used:

$$\frac{\Phi_{m,-i}}{\Phi_m} = \sum_{j \in \mathcal{I}_m \setminus \{i\}} \frac{T_j \overline{w}_j^{\varepsilon}}{\Phi_m} = \sum_{j \in \mathcal{I}_m \setminus \{i\}} s_{j|m} = 1 - s_{i|m}.$$

Plugging in the expressions for the elasticities (S1.3) and (S1.4) we get:

$$\frac{dG_1}{dw_i} = \frac{\Phi_m^{1/\varepsilon}}{w_i} \left[\psi_i (1 - s_{i|m})^{1/\varepsilon} \frac{1}{\delta} \left(\frac{1 + \delta e_i}{1 + e_i} \right) - s_{i|m} \frac{1}{\delta} \left(\frac{1 - \delta}{1 + e_i} \right) \right].$$

To obtain the elasticity, we multiply both sides by $\frac{w_i}{G_i}$:

$$\frac{dG_1}{dw_i} \frac{w_i}{G_1} = \frac{1}{1 - \psi_i (1 - s_{i|m})^{1/\varepsilon}} \left[\psi_i (1 - s_{i|m})^{1/\varepsilon} \frac{1}{\delta} \left(\frac{1 + \delta e_i}{1 + e_i} \right) - s_{i|m} \frac{1}{\delta} \left(\frac{1 - \delta}{1 + e_i} \right) \right],$$

where we used

$$\frac{\Phi_m^{1/\varepsilon}}{G_1} = \frac{\Phi_m^{1/\varepsilon}}{(\Phi_m)^{1/\varepsilon} - \psi_i\left(\Phi_{m,-i}\right)^{1/\varepsilon}} = \frac{1}{1-\psi_i\left(\Phi_{m,-i}/\Phi_m\right)^{1/\varepsilon}} = \frac{1}{1-\psi_i(1-s_{i|m})^{1/\varepsilon}}.$$

Now we need to calculate the second part of the elasticity $\frac{d\log G_i}{d\log w_i}$:

$$\frac{d\log L^s(\overline{w}_i)}{d\log w_i} = \underbrace{\frac{\partial \log L^s(\overline{w}_i)}{\partial \log \overline{w}}}_{e_i} \frac{d\log \overline{w}_i}{d\log w_i} = -\frac{1}{\delta}\left(\frac{1-\delta}{1+e_i}\right)e_i$$

Putting everything together we get:

$$\begin{aligned}
\frac{d\log G_i}{d\log w_i} &= \frac{1}{1-\psi_i(1-s_{i|m})^{1/\varepsilon}}\left[\psi_i(1-s_{i|m})^{1/\varepsilon}\frac{1}{\delta}\left(\frac{1+\delta e_i}{1+e_i}\right) - s_{i|m}\frac{1}{\delta}\left(\frac{1-\delta}{1+e_i}\right)\right] - \frac{1}{\delta}\left(\frac{1-\delta}{1+e_i}\right)e_i \\
&= \frac{1}{\delta}\frac{1}{1-\psi_i(1-s_{i|m})^{1/\varepsilon}}\left[\psi_i(1-s_{i|m})^{1/\varepsilon}\left(\frac{1+\delta e_i}{1+e_i}\right) - s_{i|m}\left(\frac{1-\delta}{1+e_i}\right) - \left(1-\psi_i(1-s_{i|m})^{1/\varepsilon}\right)\left(\frac{1-\delta}{1+e_i}\right)e_i\right] \\
&= \frac{1}{\delta}\frac{1}{1-\psi_i(1-s_{i|m})^{1/\varepsilon}}\left[\psi_i(1-s_{i|m})^{1/\varepsilon}\left(\frac{1+\delta e_i+e_i-\delta e_i}{1+e_i}\right) - \left(s_{i|m}+e_i\right)\left(\frac{1-\delta}{1+e_i}\right)\right] \\
&= \frac{1}{\delta}\frac{1}{1-\psi_i(1-s_{i|m})^{1/\varepsilon}}\left[\psi_i(1-s_{i|m})^{1/\varepsilon} - \left(s_{i|m}+e_i\right)\left(\frac{1-\delta}{1+e_i}\right)\right].
\end{aligned}$$

Substituting this into the Nash efficiency condition (S1.5) we get:

$$\frac{\varphi}{1-\psi_i(1-s_{i|m})^{1/\varepsilon}}\left[\psi_i(1-s_{i|m})^{1/\varepsilon} - \left(s_{i|m}+e_i\right)\left(\frac{1-\delta}{1+e_i}\right)\right] = (1-\varphi)(1-\delta)$$

Rearranging and factorizing terms we get:

$$\psi_i(1-s_{i|m})^{1/\varepsilon}\left(\varphi+(1-\varphi)(1-\delta)\right) = (1-\varphi)(1-\delta) + \varphi(1-\delta)\left(\frac{s_{i|m}+e_i}{1+e_i}\right).$$

Finally, solving for ψ_i we get:

$$\psi_i = \frac{1}{(1-s_{i|m})^{1/\varepsilon}}\left[(1-\omega_\psi) + \omega_\psi(1-\delta)\frac{(s_{i|m}+e_i)}{1+e_i}\right],$$

where:

$$\omega_\psi = \frac{\varphi}{\varphi + (1-\varphi)(1-\delta)}.$$

When $s_{i|m} = 0$:

$$\psi_i = \omega_\psi (1-\delta)\frac{e_i}{1+e_i} + (1-\omega_\psi) < 1$$

When $s_{i|m} \to 1$, ψ_i goes to infinity. That is why we have to impose a bound, and the expression above corresponds to $\widetilde{\psi}_i$ in the main text and the rationing share is $\min\{\widetilde{\psi}_i, 1\}$.

S1.4 Illustration

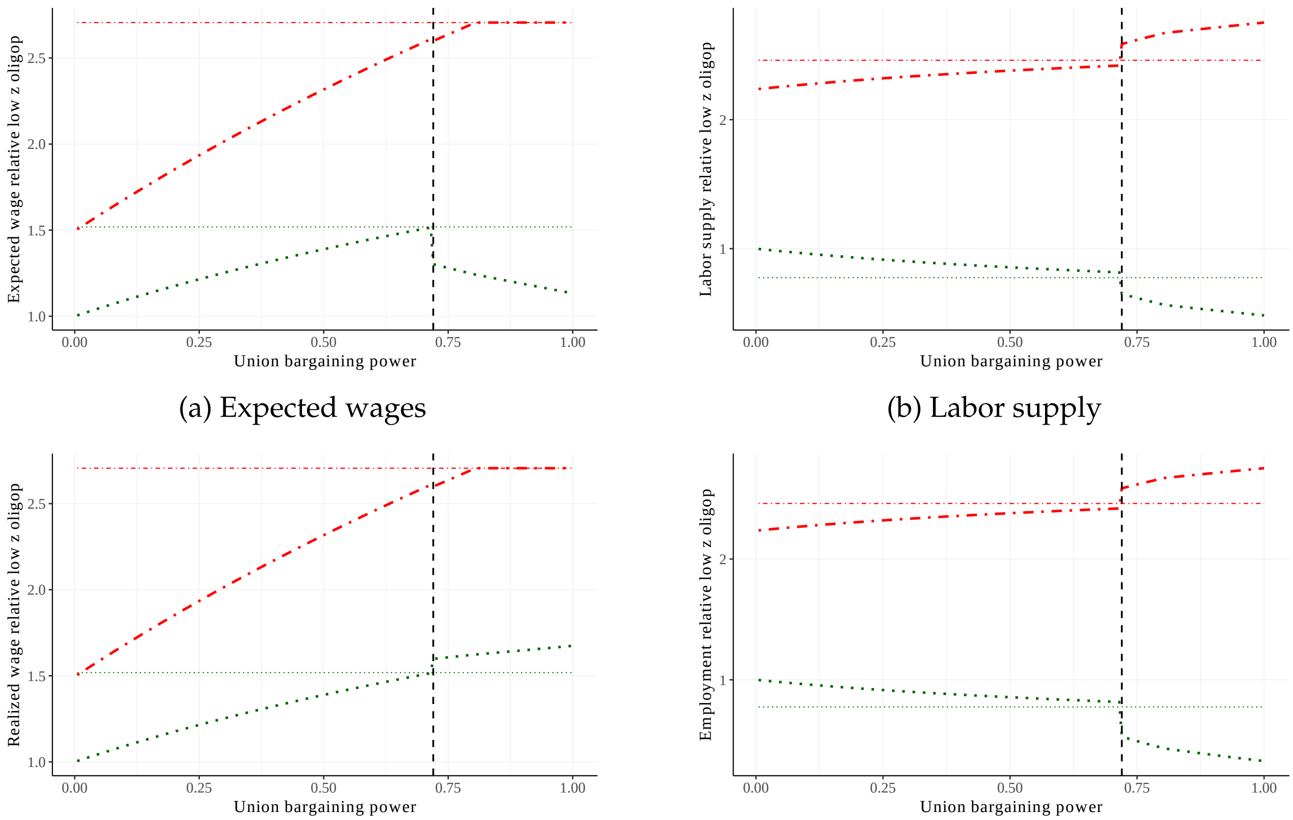
Figure S1.1 shows a comparative static of increasing the union bargaining power in a local labor market where there are two employers and unions have misaligned objectives.[1] The productive firm, denoted by firm 1, is represented by dashed red and the low productivity one, denoted by firm 2, with green dots.

The top panels present expected wages and equilibrium labor supplies to each employer. The bottom panels show the paid wages (equal to the expected ones when $\psi_i = 1$) and labor demand allocation. The horizontal dotted lines correspond to the perfect competition allocation with $\psi_i = \widetilde{\lambda}_i = 1\, \forall i \in \mathcal{I}_m$. Panel (a) of Figure S1.1 shows that for low union bargaining powers, both firms pay wages below the competitive ones as $\lambda_i < 1$ for both. Comparing both firms, we see that the productive firm (that has a higher labor supply share) is paying wages further away from the competitive ones. As a consequence, Panel (b) shows that the firm with lowest productivity attracts too many workers. As φ_b increases, before hitting the vertical green line that indicates rationing of firm 2, both firms approach the competitive wages and labor supplies (and demands).

For union bargaining powers that would bring the wage of firm 2 above the marginal revenue product of labor (beyond the vertical green line), the wages paid keep increasing in Panel (c) but the rationing decreases expected wages in Panel (a). This reduction of expected wages causes a drop of labor supply and a further drop of labor demand of firm 2 that is rationing.

[1] The local labor market corresponds to Figure 2b in Section 2.9 of the main text.

Figure S1.1: Misaligned utilities: Expected wages vs. paid wages



(a) Expected wages

(b) Labor supply

(c) Rationed wages

(d) Labor demand

Notes: Comparative statics increasing the union bargaining power φ from $\varphi = 0$ (oligopsony) to $\varphi = 1$ (monopoly union) in a local labor market with two employers, 1 (dashed red) and 2 (greed dots), where $A_1 = 2 \times A_2$, $T_1 = T_2$. All the variables on the y axis are normalized to the oligopsony values of firm 2. Panel (a): Expected wages of each employer $\overline{w}_i \equiv \psi_i w_i$. Panel (b): Labor supply. Panel (c): Wages paid to employed workers w_i. Panel (d): Labor demand.

S1.5 Model aggregation with rationing and hat algebra

When employer i is on the labor supply we have that $\psi_i = 1$ and $\lambda_i = \min\{\widetilde{\lambda}_i, 1\} = \widetilde{\lambda}_i$. Then, the wage is equal to the expected wage:

$$
\begin{aligned}
w_i &= \beta_b \lambda_i Z_i \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{-\delta} \\
\overline{w_i} &= \beta_b \lambda_i Z_i \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{-\delta} \\
&= \beta_b \lambda_i Z_i \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{S^{-\delta}}.
\end{aligned}
$$

When the employer is on the labor demand with $\psi_i \leq 1$

$$\begin{aligned} w_i &= \beta_b Z_i \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{-\delta} \\ \overline{w_i} &= \psi_i w_i = \beta_b \psi_i Z_i \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{-\delta} \\ &= \beta_b Z_i \psi_i^{1-\delta} \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{S^{-\delta}}. \end{aligned}$$

Define:

$$\overline{Z}_i := Z_i \psi_i^{1-\delta}.$$

Then noting that $\lambda_i = 1$ for the labor demand constrained firms, we have that the expected wage for all i is:

$$\overline{w}_i = \beta_b \lambda_i \overline{Z}_i \widehat{P}_b^{\frac{1}{1-\alpha_b}} L_i^{S^{-\delta}}. \tag{S1.6}$$

Substituting the labor supply from (8), $L_i^S = \frac{T_i \overline{w}_i^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L^S$, into the expected wage (S1.6), the labor supply shares are:

$$s_{i|m} = \frac{\left(T_i^{\frac{1}{\varepsilon_b}} \lambda_i \overline{Z}_i\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}{\sum_{j \in \mathcal{I}_m} \left(T_j^{\frac{1}{\varepsilon_b}} \lambda_j \overline{Z}_j\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}$$

Rewriting the employment counterfactual share as in Proposition 7 but with TFPRs:

$$s'_{i|m} = \frac{\left(T_i^{1/\varepsilon_b} \lambda'_i Z_i\right)^{\varepsilon_b/1+\varepsilon_b \delta}}{\sum_{j \in \mathcal{I}_m} \left(T_j^{1/\varepsilon_b} \lambda'_j Z_j\right)^{\varepsilon_b/1+\varepsilon_b \delta}} = \frac{s_{i|m} \left(\lambda'_i / \lambda_i\right)^{\varepsilon_b/1+\varepsilon_b \delta}}{\sum_{j \in \mathcal{I}_m} s_{j|m} \left(\lambda'_j / \lambda_j\right)^{\varepsilon_b/1+\varepsilon_b \delta}},$$

where we substituted the identified values for the revenue productivities $Z_i = \frac{w_i L_i^\delta}{\beta_b \lambda_i}$ and amenities $T_i = \frac{s_{i|m}}{(w_i)^{\varepsilon_b}} \left(\frac{L_m^S}{\Gamma_b^\eta}\right)^{\varepsilon_b/\eta}$. See Online Appendix C.3 for details. Therefore, it is equivalent to computing the counterfactual labor supply shares within a local labor market using the observed employment shares and wedges in the baseline, or the identified amenities and revenue productivities. We can then use the revenue productivities, which are a function of observed wages, employment levels and wedges to aggregate the counterfactual economy at the sector level.

Output. Employer i's output is:

$$\begin{aligned} y_i &= P_b^{\frac{\alpha_b}{1-\alpha_b}} Z_i L_i^{1-\delta} \\ &= P_b^{\frac{\alpha_b}{1-\alpha_b}} \overline{Z}_i L_i^{S\,1-\delta}. \end{aligned}$$

Market-level aggregation of output is:

$$\begin{aligned} Y_m &= P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i\in\mathcal{I}_m} \overline{Z}_i L_i^{S\,1-\delta} \\ &= P_b^{\frac{\alpha_b}{1-\alpha_b}} \left(\sum_{i\in\mathcal{I}_m} \overline{Z}_i s_{i|m}^{1-\delta} \right) L_m^{S\;1-\delta} \\ &= P_b^{\frac{\alpha_b}{1-\alpha_b}} \overline{Z}_m L_m^{S\;1-\delta}, \end{aligned}$$

where

$$\overline{Z}_m := \sum_{i\in\mathcal{I}_m} \overline{Z}_i s_{i|m}^{1-\delta}.$$

Aggregating to sector b:

$$\begin{aligned} Y_b &= \sum_{m\in\mathcal{M}_b} Y_m = \sum_{m\in\mathcal{M}_b} \sum_{i\in\mathcal{I}_m} y_i \\ &= P_b^{\frac{\alpha_b}{1-\alpha_b}} \left(\sum_{m\in\mathcal{M}_b} \sum_{i\in\mathcal{I}_m} \overline{Z}_i s_{i|m}^{1-\delta} s_{m|b}^{1-\delta} \right) L_b^{S\,1-\delta} \\ &= P_b^{\frac{\alpha_b}{1-\alpha_b}} \overline{Z}_b L_b^{S\,1-\delta}, \end{aligned}$$

where

$$\overline{Z}_b := \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m}\overline{Z}_i s_{i|m}^{1-\delta}s_{m|b}^{1-\delta}.$$

Counterfactuals. Consider now counterfactual wage at i for some counterfactual labor wedge λ_i' and rationing share ψ_i':

$$\begin{aligned}\overline{w}_i' &= \beta_b\lambda_i' A_i\psi_i' {P'}_b^{\frac{1}{1-\alpha_b}} {L_i^{S'}}^{-\delta}\\ &= \beta_b\lambda_i' Z_i\psi_i'\widehat{P}_b^{\frac{1}{1-\alpha_b}} {L_i^{S'}}^{-\delta}\\ &= \beta_b\lambda_i'\overline{Z}_i'\widehat{P}_b^{\frac{1}{1-\alpha_b}} {L_i^{S'}}^{-\delta},\end{aligned} \tag{S1.7}$$

where $\overline{Z}' \equiv Z_i{\psi_i'}^{1-\delta}$ takes the revenue productivity as fundamental. Counterfactual output is:

$$y_i' = \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}\overline{Z}'{L_i^{S'}}^{1-\delta}.$$

Market-level aggregation of counterfactual output is:

$$\begin{aligned}Y_m' &= \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}\sum_{i\in\mathcal{I}_m}\overline{Z}_i'{L_i^{S'}}^{1-\delta}\\ &= \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}\left(\sum_{i\in\mathcal{I}_m}\overline{Z}_i'{s_{i|m}'}^{1-\delta}\right){L_m^{S'}}^{1-\delta}\\ &= \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}\overline{Z}_m'{L_m^{S'}}^{1-\delta},\end{aligned}$$

where $\overline{Z}_m' := \sum_{i\in\mathcal{I}_m}\overline{Z}_i'{s_{i|m}'}^{1-\delta}$. Sector b output in the counterfactual is:

$$Y_b' = \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}\overline{Z}_b'{L_b^{S'}}^{1-\delta},$$

where $\overline{Z}_b' := \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m}\overline{Z}_i'{s_{i|m}'}^{1-\delta}{s_{m|b}'}^{1-\delta}$. Defining $\widehat{\overline{Z}}_b \equiv \frac{\overline{Z}_b'}{\overline{Z}_b}$, the counterfactual sector output relative to the baseline is:

$$\widehat{Y}_b = \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}\widehat{\overline{Z}}_b\widehat{L}_b^{S^{1-\delta}}. \tag{S1.8}$$

We can use the aggregate production function and the relative sector output (S1.8) to decompose the source of output gains in the counterfactual. The logarithm of the relative final output is:

$$\ln \widehat{Y} = \underbrace{\sum_{b \in \mathcal{B}} \theta_b \ln \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}_{\Delta \text{ GE}} + \underbrace{\sum_{b \in \mathcal{B}} \theta_b \ln \widehat{\overline{Z}}_b}_{\Delta \text{ Productivity}} + \underbrace{\sum_{b \in \mathcal{B}} \theta_b \ln \left(\widehat{L}_b^{S^{1-\delta}} \right)}_{\Delta \text{ Labor}} . \tag{S1.9}$$

The first term on the right hand side corresponds to the capital effects or general equilibrium effects of capital flowing to different sectors as a response to changes in relative prices. The second term, arguably the most important, represents total productivity gains (or losses) from less (or more) misallocation relative to the baseline. This term suffers the most from labor market concentration as big productive firms are shrinking, therefore reducing overall productivity. The third term corresponds to how labor is allocated across sectors.

We decompose $\ln \widehat{Y}$ and note that $\Delta Y = \widehat{Y} - 1 \approx \ln \widehat{Y}$ to compute contributions of each element. The share that comes from *Productivity* is $\frac{\sum_{b \in \mathcal{B}} \theta_b \ln \widehat{\overline{Z}}_b}{\ln Y}$.

S1.5.1 Solving the GE of counterfactuals

Here we drop the primes of the counterfactual variables to ease the notation. When necessary, we will denote the baseline variables with a subscript of 0. For example, baseline sector labor supply is denoted as: $L^S_{b,0}$.

Equation (S1.7) can be separated into two terms. First, a local labor market m constant. Second, an i specific component which is enough to characterize the local equilibrium as shown in Proposition 7. We denote this second term as:

$$\widetilde{\overline{w}}_i = \left(\beta_b \lambda_i \frac{\overline{Z}_i}{\left(T_i \Gamma_b^{\eta}\right)^{\delta}} \right)^{\frac{1}{1+\varepsilon_b \delta}}, \tag{S1.10}$$

where $\widetilde{\overline{w}}_i$ is a function of the labor supply shares of all the establishment-occupations i in m. The expected wage in the counterfactual is: $\overline{w}_i = \widetilde{\overline{w}}_i \Phi_m^{(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b \delta}} \widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b \delta)}} \left(\frac{\Phi}{L^S} \right)^{\frac{\delta}{1+\varepsilon_b \delta}}$. Using the definition of $\Phi_m := \sum_{i\in\mathcal{I}_m} T_i \overline{w}_i^{\varepsilon_b}$:

$$\Phi_m = \widetilde{\Phi}_m^{\frac{1+\varepsilon_b\delta}{1+\eta\delta}} \widehat{P}_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\varepsilon_b\delta}{1+\eta\delta}}, \quad \widetilde{\Phi}_m \equiv \sum_{i\in\mathcal{I}_m} T_i \widetilde{\overline{w}}_i^{\varepsilon_b}, \tag{S1.11}$$

where $\widetilde{\Phi}_m$ is a function of the local labor market equilibrium $\{s_{i|m}\}_{i\in\mathcal{I}_m}$ that can be solved separated from aggregates as shown in Proposition 7. Plugging Φ_m into the expected wage,

$$\overline{w}_i = \widetilde{\overline{w}}_i \widetilde{\Phi}_m^{\frac{(\varepsilon_b-\eta)\delta}{\varepsilon_b(1+\eta\delta)}} \widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi}{L^S}\right)^{\frac{\delta}{1+\eta\delta}}. \tag{S1.12}$$

The establishment-occupation labor supply is $L_i^S = s_{i|m}s_{m|b}L_b^S$. Given the $\widetilde{\overline{w}}_i$ we can compute the labor supply share within the local labor market and the share of m out of the sector labor supply using the definition of $\Phi_b := \sum_{m\in\mathcal{M}_b} \Phi_m^{\eta/\varepsilon_b}$ and (S1.11):

$$s_{i|m} = \frac{T_i\overline{w}_i^{\varepsilon_b}}{\Phi_m} = \frac{T_i\widetilde{\overline{w}}_i^{\varepsilon_b}}{\widetilde{\Phi}_m}, \quad \widetilde{\Phi}_m \equiv \sum_{i\in\mathcal{I}_m} T_i\widetilde{\overline{w}}_i^{\varepsilon_b},$$

$$s_{m|b} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_b} = \frac{\widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta\delta)}}}{\widetilde{\Phi}_b}, \quad \widetilde{\Phi}_b := \sum_{m\in\mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta\delta)}},$$

where $\mathcal{M}_b$ is the set of all local labor markets that belong to sector b. Knowing the relative wages within a sector, we can compute the measure of workers that go to each establishment, conditioning on sector employment. Using (S1.11), sector labor supply in the counterfactual is a function of aggregators of 'tilde' variables $\widetilde{\Phi}_b(\mathbf{s}_b)$, where $\mathbf{s}_b \equiv \{s_{i|m}\}_{i\in\mathcal{I}_b}$, and prices:

$$L_b^S = \frac{\Phi_b\Gamma_b^\eta}{\sum_{b'\in\mathcal{B}}\Phi_{b'}\Gamma_{b'}^\eta}L^S = \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}}\widetilde{\Phi}_b(\mathbf{s}_b)\Gamma_b^\eta}{\widetilde{\Phi}}L^S, \quad \widetilde{\Phi} := \sum_{b'\in\mathcal{B}} \widehat{P}_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta\delta)}}\widetilde{\Phi}_{b'}(\mathbf{s}_{b'})\Gamma_{b'}^\eta. \tag{S1.13}$$

This is where the simplifying assumption on the labor demand elasticity $\delta \equiv 1 - \frac{\beta_b}{1-\alpha_b}$ being constant across industries buys us tractability. We can factor out the economy wide constant from (S1.11) and leave everything in terms of 'tilde' variables and transformed prices.

Sector labor supply in the counterfactual relative to the baseline is: $\widehat{L}_b^S = \frac{L_b^S}{L_{b,0}}$, where $L_{b,0}$ is the baseline labor supply to b. Plugging (S1.13) into (S1.8), we have that the counterfactual sector revenue relative to the baseline is:

$$\begin{aligned}\widehat{P}_b\widehat{Y}_b &= \widehat{P}_b^{\frac{1}{1-\alpha_b}}\widehat{\overline{Z}}_b\widehat{L}_b^{S^{1-\delta}} \\ &= \widehat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}}\widehat{\overline{Z}}_b\left(\frac{\widetilde{\Phi}_b(\mathbf{s}_b)\Gamma_b^{\eta}}{L_{b,0}^S}\right)^{1-\delta}\left(\frac{L^S}{\widetilde{\Phi}}\right)^{1-\delta}.\end{aligned}$$

Finding equilibrium allocations requires solving the transformed prices relative to the baseline $\widehat{\mathbf{P}} = \{\widehat{P}_b\}_{b=1}^{\mathcal{B}}$. Using the intermediate input demand (6) and canceling constants:

$$\widehat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}}\widehat{\overline{Z}}_b\left(\frac{\widetilde{\Phi}_b\Gamma_b^{\eta}}{L_{b,0}^S}\right)^{1-\delta} = \prod_{b'\in\mathcal{B}}\left(\widehat{\overline{Z}}_{b'}\left(\frac{\widetilde{\Phi}_{b'}\Gamma_{b'}^{\eta}}{L_{b',0}^S}\right)^{1-\delta}\right)^{\theta_{b'}}\prod_{b'\in\mathcal{B}}\left(\widehat{P}_{b'}^{\frac{\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}}\right)^{\theta_{b'}}. \tag{S1.14}$$

Solving for $\widehat{P}_b$ in (S1.14):

$$\widehat{P}_b = \widehat{X}_b\widehat{X}^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}},\ \widehat{X}_b = \left(\frac{L_{b,0}^{S}{}^{1-\delta}}{\widehat{\overline{Z}}_b(\widetilde{\Phi}_b\Gamma_b^{\eta})^{1-\delta}}\right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}},\ \widehat{X} = \left(\prod_{b'\in\mathcal{B}}\widehat{X}_{b'}^{-\theta_{b'}}\right)^{\frac{1+\eta}{(1+\eta\delta)\sum_{b'\in\mathcal{B}}\theta_{b'}(1-\alpha_{b'})}}, \tag{S1.15}$$

for all $b \in \mathcal{B}$. This is analogous to the expressions in Proposition 3 but with hat variables and canceling constants.

S1.5.2 Labor share, expected wages and average wages

The establishment wage bill is: $w_iL_i = \overline{w}_i\frac{L_i}{\psi} = \overline{w}_iL_i^S = \beta_b\lambda_iP_by_i$. Aggregating to m:

$$\begin{aligned}\sum_{i\in\mathcal{I}_m} w_iL_i &= \beta_b\sum_{i\in\mathcal{I}_m}\lambda_iP_by_i = \beta_b\sum_{i\in\mathcal{I}_m}\lambda_i\frac{P_by_i}{P_bY_m}P_bY_m = \beta_b\lambda_mP_bY_m, \\ \lambda_m &\equiv \sum_{i\in\mathcal{I}_m}\lambda_i\frac{\overline{Z}_i}{\overline{\overline{Z}}_m}s_{i|m}^{1-\delta},\end{aligned}$$

where λ_m is a value added weighted sum of λ_i. Aggregating to the sector,

$$\begin{aligned}\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} w_iL_i &= \beta_b\sum_{m\in\mathcal{M}_b}\lambda_m\frac{P_bY_m}{P_bY_b}P_bY_b = \beta_b\lambda_bP_bY_b, \\ \lambda_b &\equiv \sum_{m\in\mathcal{M}_b}\lambda_m\frac{\overline{\overline{Z}}_m}{\overline{\overline{Z}}_b}s_{m|b}^{1-\delta} = \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m}\lambda_i\frac{\overline{Z}_i}{\overline{\overline{Z}}_b}s_{i|m}^{1-\delta}s_{m|b}^{1-\delta}.\end{aligned}$$

Labor demand at the sector:

$$
\begin{aligned}
L_b^D &= \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} L_i = \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \psi_i L_i^S \\
&= \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \psi_i \, s_{i|m} \, s_{m|b} \, L_b^S \\
&= (1-u_b) L_b^S,
\end{aligned}
$$

where $(1-u_b) \equiv \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \psi_i \, s_{i|m} \, s_{m|b}$, and u_b is the unemployment rate in b. The expected wage at the sector is:

$$
\begin{aligned}
\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \overline{w}_i L_i^S = \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \overline{w}_i s_{i|m} s_{m|b} L_b^S &= \overline{W}_b L_b^S = \overline{W}_b \frac{L_b^D}{(1-u_b)} = W_b L_b^D \\
\overline{W}_b &:= \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \overline{w}_i s_{i|m} s_{m|b} \\
W_b &:= \frac{\overline{W}_b}{(1-u_b)} = \frac{\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \overline{w}_i s_{i|m} s_{m|b}}{\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \psi_i \, s_{i|m} \, s_{m|b}}
\end{aligned}
$$

The labor share at b:

$$
\frac{\overline{W}_b L_b^S}{P_b Y_b} = \beta_b \lambda_b
$$

S1.6 Fixed labor

Fixing employment at the sector level b, the counterfactual expected wage (C.6) becomes:

$$
\overline{w}_i' = \left(\beta_b \lambda_i \frac{\overline{Z}_i'}{T_i^\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} \widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \Phi_m'^{\,(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b\delta}} \left(\frac{\Phi_b'}{L_b'}\right)^{\frac{\delta}{1+\varepsilon_b\delta}} .
$$

Fixing lower levels than b would only change the last element. Keeping total employment at the local labor market fixed, the last term would become: $\left(\Phi_m'/L_m'\right)^{\delta/1+\varepsilon_b\delta}$. The constant Γ_b does not appear because workers can't move across sectors. Fixing lower levels than b clearly implies that L_b' is equal to L_b. Given that L_b' is known we have a condition similar to (**??**):

$$\widehat{P}_b^{\frac{1}{1-\alpha_b}}\widehat{\widehat{Z}}_b = \prod_{b'\in\mathcal{B}}\left(\widehat{P}_{b'}^{\frac{\alpha_{b'}}{1-\alpha_{b'}}}\widehat{\widehat{Z}}_{b'}\right)^{\theta_{b'}}.$$

Propositions 8 and 4 also apply in hat counterfactuals with fixed labor at b or lower levels.

S2 Extensions

S2.1 Extension: Endogenous participation

Here we abstract from rationing or slack labor supply as we will assume that the union objective is equal to the total expected utility. Therefore $L_i = L_i^S$. We incorporate the option of being out-of-the-labor-force (from now on OTLF) by defining a new (3-digit) industry for each (2-digit) sector. These new industries have only one "employer", indexed by u, per commuting zone that 'employ' different occupations paying them a home production wage w_{uo}. The establishment-occupations define a new set of local labor markets $\mathcal{M}_u$ (combinations of commuting zones, occupations, and the new industries).

Similar to the baseline model, we assume that workers face idiosyncratic shocks that have the same Fréchet distributions. The number of workers OTLF in a particular commuting zone-sector u and occupation o is: $L_{uo} = \frac{(T_{uo}w_{uo}^{\varepsilon_b})^{\eta/\varepsilon_b}\Gamma_b^{\eta}}{\Phi}L^S$, $\Phi \equiv \Phi_e + \Phi_u$. L is the total labor supply of employed and OTLF workers. Φ is the aggregate outside option that now formed of two components: Φ_e coming from the outside options of the employed workers and Φ_u from the outside options out of the labor force. We use commuting zone level unemployment rates as proxies for OTLF rates.[2]

We assume that the OTLF rate is the same across industries and occupations in each commuting zone and define the proportion of workers OTLF in each local labor market uo accordingly. The proportion of OTLF workers in each local market identifies the home production amenity and income $T_{uo}w_{uo}^{\varepsilon_b}$ which are fixed in the counterfactuals.

The proof of Proposition 4 showed that the solution of sector prices $\mathbf{P}$ is homogeneous of

[2]We lack data on the geographical distribution of OTLF status at the commuting zone. Basing our counterfactuals in those surveys would require the assumption of constant rates of labor participation for entire regions.

degree zero with respect to total employment level which we denote here as L_e. We have that,

$$L_i(w_i) = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi} L^S = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi_e} L_e.$$

We have that $L_e = \frac{\Phi_e}{\Phi} L^S$ with $\Phi_e \equiv \sum_{m \in \mathcal{M}} \Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta$ is the part of Φ that comes from the employed and $\Phi_u \equiv \sum_{uo \in \mathcal{M}_u} (T_{uo} w_{uo}^{\varepsilon_b})^{\eta/\varepsilon_b} \Gamma_b^\eta$ is the part from the out of the labor force as in the main text.

The model aggregation steps are the same as in Proposition 2 with the exception that L_b^S now is $L_{b,e}$. The proof is in Online Appendix A.

We normalize all the reservation wages w_{uo} to 1. We recover the out-of-the-labor-force amenities T_{uo} to match the observed unemployment rate and we can compute Φ_u. There are no markdowns for the OTLF and we set the productivities of the fictitious OTLF establishments to zero such that they do not contribute to aggregate output.

Aggregating from (S1.11) from the Online Appendix,

$$\Phi_{b,e} = \left(\frac{\Phi}{L^S}\right)^{\frac{\eta\delta}{1+\eta\delta}} \sum_{m \in \mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b)}{\varepsilon_b(1+\eta\delta)}} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} = \left(\frac{\Phi}{L^S}\right)^{\frac{\eta\delta}{1+\eta\delta}} \widetilde{\Phi}_{b,e} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \tag{S2.1}$$

$$\widetilde{\Phi}_{b,e} \equiv \sum_{m \in \mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b)}{\varepsilon_b(1+\eta\delta)}}, \qquad \Phi \equiv \Phi_e + \Phi_u,$$

and,

$$\Phi_e \equiv \left(\frac{\Phi}{L^S}\right)^{\frac{\eta\delta}{1+\eta\delta}} \sum_{b \in \mathcal{B}} \widetilde{\Phi}_{b,e} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \Gamma_b^\eta = \left(\frac{\Phi}{L^S}\right)^{\frac{\eta\delta}{1+\eta\delta}} \widetilde{\Phi}_e \tag{S2.2}$$

$$\widetilde{\Phi}_e \equiv \sum_{b \in \mathcal{B}} \widetilde{\Phi}_{b,e} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \Gamma_b^\eta$$

$$L_{b,e} = \frac{\Phi_{b,e} \Gamma_b^\eta}{\Phi_e} L_e = \frac{\widetilde{\Phi}_{b,e} \Gamma_b^\eta P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}}}{\widetilde{\Phi}_e} L_e. \tag{S2.3}$$

We can solve for the prices without knowing total employment level L_e. Total employment level is $L_e = \frac{\Phi_e}{\Phi} L^S$, where L^S is total labor supply (employed and out-of-the-labor-force) that will determine the level of aggregate output. We can find it by solving for Φ_e in equation (S2.2),

$$\Phi_e^{\frac{1+\eta\delta}{\eta\delta}} L^S = (\Phi_e + \Phi_u)\widetilde{\Phi}_e^{\frac{1+\eta\delta}{\eta\delta}}.$$

The solution is unique as the left hand side is convex and equals zero when $\Phi_e = 0$, and the right hand side linear and is strictly positive when $\Phi_e = 0$. With the solution for Φ_e one can construct all the aggregates back.

S2.2 Extension: Agglomeration

Here we abstract from rationing or slack labor supply as we will assume that the union objective is equal to the total expected utility. We assume that the productivity is: $\widehat{A}_i = \widetilde{A}_i L_m^{\gamma(1-\alpha_b)}$. The agglomeration effect is a local labor market externality with elasticity $\gamma(1-\alpha_b)$. The wage first order condition is:

$$w_i = \beta_b \lambda_i Z_i L_i^{-\delta} L_m^{\gamma}. \tag{S2.4}$$

Similarly to the baseline counterfactual, we back out the transformed TFPRs, Z_i, to match observed establishment-occupation wages, w_i, under the assumption of agglomeration externalities. In the case where employment for a given local labor market is high, the backed out productivity of the establishments in that market m is lower than for the main counterfactual.

Plugging the labor supply into (S2.4), the wage in the baseline economy is:

$$w_i = \left(\beta_b \lambda_i \frac{Z_i}{(T_i \Gamma_b^{\eta})^{\delta}}\right)^{\frac{1}{1+\varepsilon_b \delta}} \Phi_m^{\nu_b - \frac{\eta}{\varepsilon_b}\widetilde{\nu_b}} \left(\frac{\Phi}{L^S}\right)^{\widetilde{\nu_b}}, \quad \nu_b = \frac{\delta}{1+\varepsilon_b \delta}, \quad \widetilde{\nu_b} = \frac{\delta - \gamma}{1+\varepsilon_b \delta}.$$

The baseline wage can be written as: $w_i = \widetilde{w}_i \Phi_m^{\nu_b - \frac{\eta}{\varepsilon_b}\widetilde{\nu_b}} \left(\frac{\Phi}{L^S}\right)^{\widetilde{\nu_b}}$. Analogously, the counterfactual wage is: $w_i = \omega_i \widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \Phi_m^{\nu_b - \frac{\eta}{\varepsilon_b}\widetilde{\nu_b}} \left(\frac{\Phi}{L^S}\right)^{\widetilde{\nu_b}}$. Aggregating to generate Φ_m,

$$\Phi_m = \widetilde{\Phi}_m^{\frac{1+\varepsilon_b\delta}{1+\eta(\delta-\gamma)}} \left(\frac{\Phi}{L^S}\right)^{\frac{\varepsilon_b(\delta-\gamma)}{1+\eta(\delta-\gamma)}}. \tag{S2.5}$$

The counterfactual Φ'_m is analogously $\Phi'_m = \widetilde{\Phi'}_m^{\frac{1+\varepsilon_b\delta}{1+\eta(\delta-\gamma)}} \widehat{P}_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} \left(\frac{\Phi}{L^S}\right)^{\frac{\varepsilon_b(\delta-\gamma)}{1+\eta(\delta-\gamma)}}$.

In order to be able to find a solution to the model, we need that the exponents are bounded. This is equivalent to requiring $\gamma \neq \frac{1}{\eta} + \delta$. The parameter γ governs the strength of agglomeration forces within a local labor market, and δ and $\frac{1}{\eta}$ are related with dispersion forces. Those come from the decreasing returns to scale (δ) and from the variance of taste shocks ($\frac{1}{\eta}$). When the latter is high, the mass of workers having extreme taste shocks is higher. This implies that agglomeration forces will impact less as workers would be more inelastic to changes in wages. The standard condition for uniqueness of the equilibrium with agglomeration would be that it is sufficiently weak ($\gamma < \frac{1}{\eta} + \delta$). We instead find the weaker condition $\gamma \neq \frac{1}{\eta} + \delta$.

The counterfactual industry labor supply is:

$$L'_b = \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} \widetilde{\Phi}'_b \Gamma_b^{\eta}}{\sum_{b\in\mathcal{B}} \widehat{P}_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta(\delta-\gamma))}} \widetilde{\Phi}'_{b'} \Gamma_{b'}^{\eta}} L^S, \quad \widetilde{\Phi}'_b \equiv \sum_{m\in\mathcal{M}_b} \widetilde{\Phi'}_m^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta(\delta-\gamma))}}$$

The counterfactual establishment-occupation output y'_i and sector output Y'_b are:

$$y'_i = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} Z_i {L'_i}^{1-\delta} {L'_m}^{\gamma}, \quad Y'_b = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} \widetilde{Z}_b {L'_b}^{1-\delta+\gamma},$$

where γ changed the returns to scale and the aggregations. We define:

$$\widetilde{Z}'_b \equiv \sum_{i\in\mathcal{I}_b} Z_i {s'_{i|m}}^{1-\delta} {s'_{m|b}}^{1-\delta+\gamma},$$

where $\widetilde{Z}'_b$ is a measure of sector counterfactual productivity with agglomeration.

The expressions for the baseline are analogous but setting $\widehat{P}_b = 1$, and defining the above with baseline employment shares, $Y'_b = \frac{1}{P_b} \widetilde{Z}'_b {L_b}^{1-\delta+\gamma}$.

The intermediate good demand in the counterfactual relative to the baseline is:

$$\widehat{P}_b^{\frac{1}{1-\alpha_b}}\widehat{\widetilde{Z}}_b\left(\frac{L'_b(\widehat{\mathbf{P}})}{L_b}\right)^{1-\delta+\gamma}=\prod_{b'\in\mathcal{B}}\left(\widehat{P}_b^{\frac{\alpha_{b'}}{1-\alpha_{b'}}}\widehat{\widetilde{Z}}_{b'}\left(\frac{L'_{b'}(\widehat{\mathbf{P}})}{L_{b'}}\right)^{1-\delta+\gamma}\right)^{\theta_{b'}}$$

$$\Leftrightarrow\ \widehat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta(\delta-\gamma))}}\widehat{\widetilde{Z}}_b\left(\frac{\widetilde{\Phi}'_b\Gamma_b^{\eta}}{L_b}\right)^{1-\delta+\gamma}=\prod_{b'\in\mathcal{B}}\left(\widehat{P}_{b'}^{\alpha_{b'}(1+\eta(\delta-\gamma))+\eta(1-\delta+\gamma)}\widehat{\widetilde{Z}}_{b'}\left(\frac{\widetilde{\Phi}'_{b'}\Gamma_{b'}^{\eta}}{L_{b'}}\right)^{1-\delta+\gamma}\right)^{\theta_{b'}}.$$

Uniqueness of the solution to this system of equations is guaranteed by $\sum_{b\in\mathcal{B}}\alpha_b\theta_b<1$. This condition being the same as for Proposition 4, uniqueness of the equilibrium with agglomeration forces only needs the additional requirement of $\gamma\neq\frac{1}{\eta}+\delta$.

S3 Additional results and proofs

We use the following Theorem and Corollary to establish uniqueness in our proofs. These are taken from Allen, Arkolakis, and Li (2016) as they are not present any more in the current version of their paper Allen, Arkolakis, and Li (2023). Of course, any error should be attributed to us.

Theorem 1. *Consider $g:\mathbb{R}^n_{++}\times\mathbb{R}^m_{++}$ for some $n\in\{1,...,N\}$ and $m\in\{1,...,M\}$ such that:*

1. *homogeneity of any degree: $g(tx,ty)=t^k g(x,y)$, $t\in\mathbb{R}_{++}$ and $k\in\mathbb{R}$,*
2. *gross-substitution property: $\frac{\partial g_i}{\partial x_j}>0$ for all $i\neq j$,*
3. *monotonicity with respect to the joint variable: $\frac{\partial g_i}{\partial y_k}\geq 0$, for all i,k.*

Then, for any given $y^0\in\mathbb{R}^M_{++}$ there exists at most one solution satisfying $g(x,y^0)=0$.

Proof. We proceed by contradiction. Suppose there are two different up-to-scale, solutions, x^1, x^2, such that $f(x^1)=f(x^2)=0$ i.e. $g(x^1,y^0)=g(x^2,y^0)=0$. Without loss of generality, suppose there exists some $t>1$ such that $tx^1_j\geq x^2_j$ for all $j\in\{1,...,n\}$ and the equality holds for at least one $j=\bar{j}$. Then the inequality must strictly hold since x^1 and x^2 are different up-to-scale. Condition (iii) $\frac{\partial g_i}{\partial y_k}\geq 0$, for all i,k implies that $g(tx^1,y^0)\leq g(tx^1,ty^0)=0$ where $g(tx^1,ty^0)=0$ is from condition (i) (and also $g(tx^2,ty^0)=0$ because x^1 and x^2 are solutions). However, condition (ii) implies $g_j(tx^1,y^0)>g_j(x^2,y^0)=0$, thus a contradiction. □

Corollary 1. *Assume (i) $f(x)$ satisfies gross-substitution and (ii) $f(x)$ can be decomposed as $f(x) = \sum_{j=1}^{\nu_f} g^j(x) - \sum_{k=1}^{\nu_g} h^k(x)$, where $g^j(x), h^k(x)$ are non-negative vector functions and, respectively, homogeneous of degree α_j and β_k, with $\bar{\alpha} = \max \alpha_j \leq \min \beta_k$.*

1. *Then there is at most one up-to-scale solution of $f(x) = 0$.*

2. *In particular, if for some j, k $\alpha_j \neq \beta_k$, then there is at most one solution.*

Proof. Define $m(x, y)$ as a vector function where $m_i(x, y) = \sum_{j=1}^{\nu_f} y^{\bar{\alpha}-\alpha_j} g_i^j(x) - \sum_{k=1}^{\nu_g} y^{\bar{\alpha}-\beta_k} h_i^k(x)$. Obviously, $m(x, y)$ is of homogenous degree $\bar{\alpha}$ and $\frac{\partial m_i}{\partial y} \geq 0$. Also we have $f(x) = m(x, y^0)$ where $y^0 = 1$, thus the above theorem applies.

Furthermore, if $f_i(x)$ is not homogeneous of some degree because $\alpha_j \neq \beta_k$, there is at most one solution. Suppose not, if tx^1 and x^1 are the solutions, then $f_i(x^1) > t^{-min(\beta_k)} f_i(tx^1) = 0$, also a contradiction. □

S4 Sector wage floors

Here we present more details in the aggregate bargaining model where a union negotiates sectoral minimum wages and we explain the algorithm to solve for this model.

First, let us recall the three regions that an employer can be located. On the first one, denoted Region I, employers are not constrained and charge their oligopsony wage. The second one, Region II, employers are constrained by the minimum wage but their employment is below the labor demand at that wage, so there is no rationing. On the third region, Region III, employers are constrained by the minimum wage and by the labor demand, so there is rationing.

As we will show below, it is useful to characterize the whole system using expected wages. Naturally, for those employers in Regions I and II the expected wages are equal to the actual wages as there is no rationing. We now characterize the expected wages for the different regions.

In the following, to ease on notation clutter, we will abstract from the "prime" notation to denote the counterfactual variable. Using the TFPRs Z_i from the baseline equilibrium and the relative prices $\widehat{P}_b$ we have that the wages for employers in Region I are:

$$\overline{w}_i = \beta_b \mu_i Z_i \left(\widehat{P}_b\right)^{\frac{1}{1-\alpha_b}} \left(L_i^S\right)^{-\delta}.$$

After some algebra, we can develop this expression and get:

$$\overline{w}_i = \left[\beta_b \mu_i Z_i T_i^{-\frac{\delta\eta}{\varepsilon_b}}\right]^{\frac{1}{1+\delta\eta}} \left(s_{i|m}\right)^{\frac{\delta(\eta-\varepsilon_b)}{\varepsilon_b(1+\delta\eta)}} \left(\widehat{P}_b\right)^{\frac{1}{(1-\alpha_b)(1+\delta\eta)}} \left(\frac{\Phi}{\Gamma_b^\eta L^S}\right)^{\frac{\delta}{1+\delta\eta}}. \tag{S4.1}$$

For employers in Region II the expected wage is, trivially:

$$\overline{w}_i = \underline{w}_b. \tag{S4.2}$$

For employers in Region III is a little more complicated as the rationing share ψ_i and expected wage $\overline{w}_i = \psi_i \underline{w}_b$ are endogenous. These employers would equalize their marginal revenue product to the minimum wage. Multiplying by the rationing share on the left and right hand side of that expression and substituting labor demand to labor supply using the definition of the rationing share we get:

$$\overline{w}_i = \beta_b (\psi_i)^{1-\delta} Z_i \left(\widehat{P}_b\right)^{\frac{1}{1-\alpha_b}} \left(L_i^S\right)^{-\delta}.$$

Multiplying and dividing $(\psi_i)^{1-\delta}$ by the minimum wage and developing we get

$$\overline{w}_i = \left[\beta_b (\underline{w}_b)^{\delta-1} Z_i\right]^{\frac{1}{\delta}} \left(\widehat{P}_b\right)^{\frac{1}{\delta(1-\alpha_b)}} \left(L_i^S\right)^{-1}.$$

Substituting the labor supply, $L_i^S = \left(T_i \overline{w}_i^{\varepsilon_b}\right)^{\frac{\eta}{\varepsilon_b}} \left(s_{i|m}\right)^{1-\frac{\eta}{\varepsilon_b}} \frac{\Gamma_b^\eta L^S}{\Phi}$, and developing we get:

$$\overline{w}_i = \left[\beta_b (\underline{w}_b)^{\delta-1} Z_i T_i^{-\frac{\delta\eta}{\varepsilon_b}}\right]^{\frac{1}{\delta(1+\eta)}} \left(s_{i|m}\right)^{\frac{(\eta-\varepsilon_b)}{\varepsilon_b(1+\eta)}} \left(\widehat{P}_b\right)^{\frac{1}{\delta(1-\alpha_b)(1+\eta)}} \left(\frac{\Phi}{\Gamma_b^\eta L^S}\right)^{\frac{1}{1+\eta}}. \tag{S4.3}$$

Regardless to which Region an employer belongs, their expected wage is determined by either (S4.1), (S4.2) or (S4.3), which, taking as given Φ and $\widehat{P}_b$, are functions of parameters or other expected wages.

S4.1 Algorithm to solve local labor market equilibrium

To solve for the local labor market we take as given Φ. The solution algorithm needs to determine the region of each employer and solve the system of equations. To do so we need an expression for labor demand which is:

$$\ell_i^D(w_i) = \left[\beta_b Z_i w_i^{-1}\right]^{\frac{1}{\delta}}.$$

The algorithm to solve for the equilibrium of local labor market m is as follows:

1. Initiate all employers to belong to Region I.
2. Solve equilibrium expected wages $\{\overline{w}_i\}_{i\in m}$ based on Region status using (S4.1), (S4.2) or (S4.3) for each employer i.
3. For all i that belong to Region I who have $\overline{w}_i < \underline{w}_b$ plus all i that belong to Region II and III do:
   (a) Evaluate $\ell_i^D(\underline{w}_b)$ and $\ell_i^S(\underline{w}_b)$, where the other employer wages for the labor supply of i are those obtained in step 2.
   (b) If $\ell_i^D(\underline{w}_b) > \ell_i^S(\underline{w}_b)$, then i belongs to Region II.
   (c) Otherwise, i belongs to Region III.
4. Check is there is some i that changed Region. If so, repeat steps 2 to 4. If not, end.

S4.2 Algorithm to solve for general equilibrium

Here we detail how to close the algorithm to solve for the general equilibrium.

1. Given the initial value of $\Phi^{(0)}$ solve for local labor market equilibrium for all m.
2. Use solution of wages to derive labor supplies, and for those employers in Region III their labor demands.
3. Compute rationing shares ψ_i for employers in Region III and labor supply shares $s_{i|m}$.

4. Aggregate using ψ_i and $s_{i|m}$.

5. Solve for the new economy-wide constant: $\Phi^{(1)}$.

6. Check if $\Phi^{(1)}$ is close to $\Phi^{(0)}$. If so, end. If not, update $\Phi^{(0)}$ and go back to step 1.

S4.3 Bargained sector wage floors

We can consider two objective functions for the sector-wide union: (i) sector welfare $\Phi_b = \sum_{m\in\mathcal{M}_b} \Phi_m^{\eta/\varepsilon_b}$ or the total wage bill. We assume that the sector-wide union's objective is the sector wage bill.

S5 Representative household formulation

To derive the representative household formulation, we build from entropy-based derivations of gravity-type models (Wilson (2010)). We show that this formulation leads to the same labor supply functions and equivalent welfare measures.

We consider a representative household who chooses where each worker within the household works and how much they consume. The household gets disutility when choosing where to allocate workers according to a nested entropy function. Furthermore, the household faces different budget constraints which say that total consumption of workers employed with an employer has to be less or equal than the income those workers receive. We assume that each worker receives log utility from consumption. Therefore, the representative household will equalize consumption for workers working with the same employer and each would get the expected wage $\overline{w}_i$. After substituting the budget constraints, the representative household problem is:

$$\max_{\{s_i\}_{i\in m\in\mathcal{M}}} \sum_{m\in\mathcal{M}} \sum_{i\in m} \ln(\overline{w}_i)s_i + \sum_{b\in\mathcal{B}} \frac{1}{\varepsilon_b} \sum_{m\in\mathcal{M}_b} s_m H_m(\vec{s}_m) + \frac{1}{\eta} H(\vec{s})$$

subject to:

$$\sum_{m\in\mathcal{M}} \sum_{i\in m} s_i = 1, \quad \text{and} \quad s_i \geq 0 \text{ for all } i,$$

where:

- $H_m(\vec{s}_m) = -\sum_{i\in m} s_{i|m} \ln \frac{s_{i|m}}{T_i}$ is the within-market entropy,
- $H(\vec{s}) = -\sum_{b\in\mathcal{B}} \sum_{m\in\mathcal{M}_b} s_m \ln \frac{s_m}{\Gamma_b}$ is the between-market entropy,
- s_i is the labor supply share for occupation i,
- $s_m = \sum_{i\in m} s_i$ is the labor supply share for market/nest m,
- $s_{i|m} = \frac{s_i}{s_m}$ is the conditional share of occupation i within market/nest m,
- $\vec{s}_m = \{s_{i|m} \ i \in m\}$,
- $\vec{s} = \{s_m \ m \in \mathcal{M}\}$.

We solve this problem hierarchically, first determining the conditional shares within nests, then the nest shares, and finally combining them.

Step 1: Conditional shares within nests For a given nest m in category b with share s_m, we maximize:

$$\max_{\{s_i\}_{i\in m}} \sum_{i\in m} \ln(\overline{w}_i)s_i + \frac{1}{\varepsilon_b} s_m H_m(\vec{s}_m),$$

subject to:

$$\sum_{i\in m} s_i = s_m.$$

Since $s_i = s_m \cdot s_{i|m}$, we can rewrite this as:

$$\max_{\{s_{i|m}\}_{i\in m}} \sum_{i\in m} \ln(\overline{w}_i)s_m s_{i|m} + \frac{1}{\varepsilon_b} s_m \left(-\sum_{i\in m} s_{i|m} \ln \frac{s_{i|m}}{T_i} \right),$$

Factoring out s_m, expanding the entropy and modifying the constraint we have:

$$\max_{\{s_{i|m}\}_{i\in m}} s_m \left[\sum_{i\in m} \ln(\overline{w}_i)s_{i|m} - \frac{1}{\varepsilon_b} \sum_{i\in m} s_{i|m} \ln s_{i|m} + \frac{1}{\varepsilon_b} \sum_{i\in m} s_{i|m} \ln T_i \right],$$

subject to:

$$\sum_{i \in m} s_{i|m} = 1.$$

Setting up the Lagrangian with multiplier ζ_m:

$$\mathcal{L}_m = s_m \left[\sum_{i \in m} \ln(\overline{w}_i) s_{i|m} - \frac{1}{\varepsilon_b} \sum_{i \in m} s_{i|m} \ln s_{i|m} + \frac{1}{\varepsilon_b} \sum_{i \in m} s_{i|m} \ln T_i \right] - \zeta_m \left(\sum_{i \in m} s_{i|m} - 1 \right).$$

Taking the derivative with respect to $s_{i|m}$:

$$\begin{aligned}
\frac{\partial \mathcal{L}_m}{\partial s_{i|m}} &= s_m \left[\ln(\overline{w}_i) - \frac{1}{\varepsilon_b} (\ln s_{i|m} + 1) + \frac{1}{\varepsilon_b} \ln T_i \right] - \zeta_m \\
&= s_m \ln(\overline{w}_i) - \frac{s_m}{\varepsilon_b} \ln s_{i|m} - \frac{s_m}{\varepsilon_b} + \frac{s_m}{\varepsilon_b} \ln T_i - \zeta_m = 0.
\end{aligned}$$

Rearranging:

$$\begin{aligned}
\frac{s_m}{\varepsilon_b} \ln s_{i|m} &= s_m \ln(\overline{w}_i) - \frac{s_m}{\varepsilon_b} + \frac{s_m}{\varepsilon_b} \ln T_i - \zeta_m \\
\ln s_{i|m} &= \varepsilon_b \ln(\overline{w}_i) - 1 + \ln T_i - \frac{\varepsilon_b \zeta_m}{s_m}.
\end{aligned}$$

Taking the exponential of both sides:

$$\begin{aligned}
s_{i|m} &= \exp \left(\varepsilon_b \ln(\overline{w}_i) - 1 + \ln T_i - \frac{\varepsilon_b \zeta_m}{s_m} \right) \\
&= e^{-1} \cdot e^{-\frac{\varepsilon_b \zeta_m}{s_m}} \cdot (\overline{w}_i)^{\varepsilon_b} \cdot T_i \\
&= \mathcal{K}_m \cdot (\overline{w}_i)^{\varepsilon_b} \cdot T_i,
\end{aligned}$$

where $\mathcal{K}_m = e^{-1} \cdot e^{-\frac{\varepsilon_b \zeta_m}{s_m}}$ is a nest-specific constant. Using the constraint $\sum_{i \in m} s_{i|m} = 1$:

$$\begin{aligned}
\sum_{i \in m} \mathcal{K}_m \cdot (\overline{w}_i)^{\varepsilon_b} \cdot T_i &= 1 \\
\mathcal{K}_m \sum_{i \in m} (\overline{w}_i)^{\varepsilon_b} \cdot T_i &= 1 \\
\mathcal{K}_m &= \frac{1}{\sum_{i \in m} (\overline{w}_i)^{\varepsilon_b} \cdot T_i}.
\end{aligned}$$

Therefore, the optimal conditional shares are:

$$s_{i|m} = \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\sum_{j \in m} (\overline{w}_j)^{\varepsilon_b} \cdot T_j}$$

Step 2: Inclusive Value The maximum value of the inner optimization for market/nest m in category b is:

$$V_m = \sum_{i \in m} \ln(\overline{w}_i) s_{i|m} - \frac{1}{\varepsilon_b} \sum_{i \in m} s_{i|m} \ln \frac{s_{i|m}}{T_i}.$$

Substituting the optimal $s_{i|m}$:

$$\begin{aligned}
V_m = & \sum_{i \in m} \ln(\overline{w}_i) \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\sum_{j \in m} (\overline{w}_j)^{\varepsilon_b} \cdot T_j} \\
& - \frac{1}{\varepsilon_b} \sum_{i \in m} \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\sum_{j \in m} (\overline{w}_j)^{\varepsilon_b} \cdot T_j} \ln \left(\frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\sum_{j \in m} (\overline{w}_j)^{\varepsilon_b} \cdot T_j} \cdot \frac{1}{T_i} \right)
\end{aligned}$$

Let's denote $\Phi_m = \sum_{j \in m} (\overline{w}_j)^{\varepsilon_b} \cdot T_j$ for brevity:

$$\begin{aligned}
V_m &= \sum_{i \in m} \ln(\overline{w}_i) \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\Phi_m} - \frac{1}{\varepsilon_b} \sum_{i \in m} \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\Phi_m} \ln \left(\frac{(\overline{w}_i)^{\varepsilon_b}}{\Phi_m} \right) \\
&= \sum_{i \in m} \ln(\overline{w}_i) \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\Phi_m} - \frac{1}{\varepsilon_b} \sum_{i \in m} \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\Phi_m} \left[\varepsilon_b \ln(\overline{w}_i) - \ln(\Phi_m) \right] \\
&= \sum_{i \in m} \ln(\overline{w}_i) \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\Phi_m} - \sum_{i \in m} \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\Phi_m} \ln(\overline{w}_i) + \frac{1}{\varepsilon_b} \sum_{i \in m} \frac{(\overline{w}_i)^{\varepsilon_b} \cdot T_i}{\Phi_m} \ln(\Phi_m).
\end{aligned}$$

Since $\sum_{i\in m} \frac{(\overline{w}_i)^{\varepsilon_b}\cdot T_i}{\Phi_m} = 1$, the first two terms cancel out, and:

$$V_m = \frac{1}{\varepsilon_b}\ln(\Phi_m) = \frac{1}{\varepsilon_b}\ln\left(\sum_{i\in m}(\overline{w}_i)^{\varepsilon_b}\cdot T_i\right).$$

Step 3: Nest Shares Using V_m as the utility of each nest, we maximize:

$$\max_{\{s_m\}_{m\in\mathcal{M}}} \sum_{m\in\mathcal{M}} V_m s_m + \frac{1}{\eta}H(\vec{s}),$$

subject to:

$$\sum_{m\in\mathcal{M}} s_m = 1.$$

Expanding the entropy term:

$$\max_{\{s_m\}_{m\in\mathcal{M}}} \sum_{m\in\mathcal{M}} V_m s_m - \frac{1}{\eta}\sum_{m\in\mathcal{M}} s_m \ln s_m + \frac{1}{\eta}\sum_{b\in\mathcal{B}}\sum_{m\in\mathcal{M}_b} s_m \ln\Gamma_b.$$

Setting up the Lagrangian with multiplier ζ:

$$\mathcal{L} = \sum_{m\in\mathcal{M}} V_m s_m - \frac{1}{\eta}\sum_{m\in\mathcal{M}} s_m \ln s_m + \frac{1}{\eta}\sum_{b\in\mathcal{B}}\sum_{m\in\mathcal{M}_b} s_m \ln\Gamma_b - \zeta\left(\sum_{m\in\mathcal{M}} s_m - 1\right).$$

Taking the derivative with respect to s_m:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial s_m} &= V_m - \frac{1}{\eta}(\ln s_m + 1) + \frac{1}{\eta}\ln\Gamma_b - \zeta = 0 \\
\frac{1}{\eta}\ln s_m &= V_m - \frac{1}{\eta} + \frac{1}{\eta}\ln\Gamma_b - \zeta \\
\ln s_m &= \eta V_m - 1 + \ln\Gamma_b - \eta\zeta
\end{aligned}$$

Taking the exponential of both sides:

$$
\begin{aligned}
s_m &= \exp\left(\eta V_m - 1 + \ln \Gamma_b - \eta\zeta\right) \\
&= e^{-1} \cdot e^{-\eta\zeta} \cdot e^{\eta V_m} \cdot \Gamma_b \\
&= \mathcal{K} \cdot e^{\eta V_m} \cdot \Gamma_b,
\end{aligned}
$$

Where $\mathcal{K} = e^{-1} \cdot e^{-\eta\zeta}$ is a constant. Using the constraint $\sum_{m \in \mathcal{M}} s_m = 1$:

$$
\begin{aligned}
\sum_{m \in \mathcal{M}} \mathcal{K} \cdot e^{\eta V_m} \cdot \Gamma_b = 1 \\
\mathcal{K} \sum_{m \in \mathcal{M}} e^{\eta V_m} \cdot \Gamma_b = 1 \\
\mathcal{K} = \frac{1}{\sum_{m \in \mathcal{M}} e^{\eta V_m} \cdot \Gamma_b}.
\end{aligned}
$$

Therefore, the optimal nest shares are:

$$
s_m = \frac{e^{\eta V_m} \cdot \Gamma_b}{\sum_{n \in \mathcal{M}} e^{\eta V_n} \cdot \Gamma_n}
$$

Substituting the expression for V_m:

$$
\begin{aligned}
s_m &= \frac{e^{\eta \cdot \frac{1}{\varepsilon_b} \ln\left(\sum_{i \in m} (\overline{w}_i)^{\varepsilon_b} \cdot T_i\right)} \cdot \Gamma_b}{\sum_{b \in \mathcal{B}} \sum_{m' \in \mathcal{M}_b} e^{\eta \cdot \frac{1}{\varepsilon_b} \ln\left(\sum_{i \in m'} (\overline{w}_i)^{\varepsilon_b} \cdot T_i\right)} \cdot \Gamma_b} \\
&= \frac{\left(\sum_{i \in m} (\overline{w}_i)^{\varepsilon_b} \cdot T_i\right)^{\frac{\eta}{\varepsilon_b}} \cdot \Gamma_b}{\sum_{b \in \mathcal{B}} \sum_{m' \in \mathcal{M}_b} \left(\sum_{i \in m'} (\overline{w}_i)^{\varepsilon_b} \cdot T_i\right)^{\frac{\eta}{\varepsilon_b}} \cdot \Gamma_b}.
\end{aligned}
$$

Then, all the employment shares are exactly identical to those derived using the extreme value shocks in the main text.

S5.1 Counterfactual welfare

The maximum value of our original objective function is:

$$\frac{1}{\eta} \ln \left(\sum_{m \in \mathcal{M}} \left(\sum_{i \in m} (\overline{w}_i)^{\varepsilon_b} \cdot T_i \right)^{\frac{\eta}{\varepsilon_b}} \cdot \Gamma_b \right).$$

This function above we can take it as our welfare measure. Here we will show that constructing a consumption-equivalent measure for changes in welfare is the same as in the main text.

Say there is a counterfactual where expected wages are $\overline{w}_i'$. We want to compute how much consumption has to change, proportionally, so that the representative household is indifferent between the baseline equilibrium and the counterfactual. Now notice that the expression above is just equal to $\frac{1}{\eta} \ln \Phi$, where Φ is defined as in the main text. In other words, it is the log of our welfare measure in the main text. Naturally, we can define then the consumption-equivalent measure χ so that $\frac{1}{\eta} \ln \Phi + \ln \chi = \frac{1}{\eta} \ln \Phi'$, where Φ' is the analogous of Φ in the counterfactual equilibrium. Then $\chi = \frac{(\Phi')^{\frac{1}{\eta}}}{(\Phi)^{\frac{1}{\eta}}}$, which is exactly the same measure we use in the main text.

S6 Social welfare decomposition

In this section we show how to do the aggregate efficiency and redistribution decomposition of social welfare gains as in Berger et al. (2025).

Scaled amenities. A desirable property of what we would call "amenities" is that when we scale all of them by a multiplicative constant, workers' welfare also scales by the same constant. What we have been calling "amenities" so far, the T_i's, do not have this property. This is easy to see as workers' welfare $\Phi^{1/\eta}$ is not homogeneous in amenities. What we seek is a notion of amenity so that welfare is homogeneous of degree one with respect to them. Thus, we define a *scaled amenity*:

$$\widetilde{T}_i \equiv T_i^{1/\varepsilon_b}.$$

Recall that $\mathcal{I}$ is the set of all employers, $\mathcal{I}_m$ is the set containing all employers within market m, $\mathcal{M}_b$ is the set of all markets in sector b, and $\mathcal{B}$ is the set containing all sectors. Then, workers' welfare, written using these scaled amenities, equals:

$$\mathcal{W} = \left[\sum_{b \in \mathcal{B}} \Gamma_b^{\eta} \sum_{m \in \mathcal{M}_b} \left(\sum_{i \in \mathcal{I}_m} \left(\widetilde{T}_i w_i \right)^{\varepsilon_b} \right)^{\frac{\eta}{\varepsilon_b}} \right]^{\frac{1}{\eta}}.$$

Clearly, $\mathcal{W}$ is homogeneous of degree one with respect to the scaled amenities.

Social welfare. In our economy there are two types of agents: workers and employers. In contrast to standard macro models, we separate employers (who own the firms) from workers. We assume employers derive utility linearly from consumption, so their total utility equals total profits. Workers derive utility from both consumption *and* idiosyncratic taste shocks, which are functions of scaled amenities $\{\widetilde{T}_i\}$. Thus, different employment distributions give different utility to workers *regardless* of their total consumption.

We can treat total workers' welfare as the utility of one representative worker, and total profits as the utility of one representative employer. This is not important from a utilitarian perspective where social welfare is simply the sum of individual welfares.

Total consumption in the economy C equals

$$C = WB + \Pi,$$

where Π is total profits and WB is the total wage bill:

$$WB \equiv \sum_i w_i s_i L^S.$$

Workers' total consumption is the wage bill; employers' total consumption is total profits.

To match the notation of Berger et al. (2025) as closely as possible, we denote N as an employment index equal to:

$$N \equiv \left[\sum_{b \in \mathcal{B}} \Gamma_b^{\eta} \sum_{m \in \mathcal{M}_b} \left(\sum_{i \in \mathcal{I}_m} \widetilde{T}_i^{\varepsilon_b} \left(\frac{w_i}{\sum_{j \in \mathcal{I}} w_j s_j L^S} \right)^{\varepsilon_b} \right)^{\frac{\eta}{\varepsilon_b}} \right]^{\frac{1}{\eta}}.$$

The employment index N gives the utility workers obtain from their employment distribution across employers *net* of their aggregate consumption level WB.

Using the expression for total consumption, we obtain the consumption share for workers:

$$\sigma \equiv \frac{WB}{WB + \Pi}.$$

Then, total consumption for workers equals σC. Similarly, total consumption for employers equals $(1 - \sigma)C$.

We can then decompose workers' welfare $\mathcal{W}$ into an aggregate consumption component C, a consumption share σ, and an employment index N:

$$\mathcal{W}(\sigma, C, N) = \sigma C N.$$

Similarly, we can define employers' utility $\mathcal{W}^f$ as a function of σ and C:

$$\mathcal{W}^f(\sigma, C) = (1 - \sigma)C.$$

To make welfare comparable across workers and employers, we transform them into consumption-equivalent terms. In our case this is easy as both workers' and employers' welfare are *linear* in their total consumption. In other words, the marginal utility of consumption does not depend on consumption levels. For workers it is N, and for employers it is 1.

Use bar notation to define a reference allocation (e.g., $\overline{\sigma}$). Then, the reference employment index $\overline{N}$ equals:

$$\overline{N} \equiv \left[\sum_{b \in \mathcal{B}} \Gamma_b^{\eta} \sum_{m \in \mathcal{M}_b} \left(\sum_{i \in \mathcal{I}_m} \widetilde{T}_i^{\varepsilon_b} \left(\frac{\overline{w}_i}{\sum_{j \in \mathcal{I}} \overline{w}_j \bar{s}_j L^S} \right)^{\varepsilon_b} \right)^{\frac{\eta}{\varepsilon_b}} \right]^{\frac{1}{\eta}}.$$

For any allocation (σ, C, N), we define the consumption-equivalent workers' welfare $\widetilde{\mathcal{W}}$:

$$\widetilde{\mathcal{W}}(\sigma, C, N) = \frac{\mathcal{W}(\sigma, C, N)}{\overline{N}} = \frac{\sigma C N}{\overline{N}}.$$

Defined this way, the consumption-equivalent workers' welfare answers the following question: *How much consumption would workers need in the reference allocation to achieve the same utility as in the current allocation?* Using this measure we can now compare it with employers' utility, which is already defined in consumption-equivalent terms. Naturally, $\widetilde{\mathcal{W}}$ is invariant to proportional changes in scaled amenities. Also, $\widetilde{\mathcal{W}}(\overline{\sigma}, \overline{C}, \overline{N}) = \overline{\sigma}\overline{C}$; in other words, the consumption-equivalent workers' welfare in the reference allocation equals the wage bill.

We can now define social welfare $\mathcal{S}$ and normalized social welfare $\mathcal{S}_\Gamma$ as in Berger et al. (2025) (see pp. 291-292):

$$\mathcal{S}(\sigma, C, N) \equiv \widetilde{\mathcal{W}}(\sigma, C, N) + \mathcal{W}^f(\sigma, C), \quad \mathcal{S}_\Gamma(\sigma, C, N) \equiv \frac{\mathcal{S}(\sigma, C, N)}{\Gamma},$$

where Γ is a normalization to express differences in normalized welfare as welfare gains with respect to the reference allocation; this is useful for the decomposition below. Let $\overline{\sigma}$, $\overline{N}$, and $\overline{C}$ be, respectively, the workers' consumption share, the workers' employment index, and the economy's total consumption in a reference allocation (e.g., the oligopsony economy). Then Γ equals:

$$\Gamma \equiv \mathcal{S}(\overline{\sigma}, \overline{C}, \overline{N}).$$

Given the definition of Γ, $\mathcal{S}_\Gamma(\overline{\sigma}, \overline{C}, \overline{N}) = 1$.

Decomposition. As in the main text, denote any counterfactual allocation with primes (e.g., σ'). Then, normalized social welfare gains can be decomposed as the sum of aggregate efficiency and redistribution gains:

$$\underbrace{\mathcal{S}_\Gamma(\sigma', C', N') - \mathcal{S}_\Gamma(\overline{\sigma}, \overline{C}, \overline{N})}_{\text{Social welfare gains}} = \frac{1}{\Gamma}\left[\mathcal{S}(\sigma', C', N') - \mathcal{S}(\overline{\sigma}, \overline{C}, \overline{N})\right]$$
$$= \underbrace{\frac{1}{\Gamma}\left[\mathcal{S}(\overline{\sigma}, C', N') - \mathcal{S}(\overline{\sigma}, \overline{C}, \overline{N})\right]}_{\text{Aggregate Efficiency}} + \underbrace{\frac{1}{\Gamma}\left[\mathcal{S}(\sigma', C', N') - \mathcal{S}(\overline{\sigma}, C', N')\right]}_{\text{Redistribution}}.$$

As Berger et al. (2025) explain, aggregate efficiency captures the effects of increasing total consumption C and the employment index N, holding consumption shares fixed.[3] These gains come from increasing the overall size of the "economic pie." Redistribution gains capture the effects of shifting consumption shares, so gains come only from moving consumption from employers to workers (or vice versa). In our context, redistribution gains reflect how much welfare would increase if we left the employment distribution constant but allowed the consumption share to change.

S7 Bias of the reduced-form elasticity of labor supply

Here we do a step-by-step derivation of the bias in the reduced-form labor supply elasticity in the presence of strategic interactions.

Decomposition. We begin with the reduced-form elasticity and decompose it using the chain rule:

$$\frac{d\ln L_i}{d\ln w_i} = \frac{d\ln L_i}{d\ln\left(w_i/w_j\right)} \frac{d\ln\left(w_i/w_j\right)}{d\ln w_i}.$$

We can rewrite the first term by adding and subtracting $\ln L_j$ for any $j \neq i$:

$$\frac{d\ln L_i}{d\ln\left(w_i/w_j\right)} = \frac{d\left(\ln L_i - \ln L_j + \ln L_j\right)}{d\ln\left(w_i/w_j\right)} = \frac{d\ln\left(L_i/L_j\right)}{d\ln\left(w_i/w_j\right)} + \frac{d\ln L_j}{d\ln\left(w_i/w_j\right)}.$$

[3] In our framework we do not have the equivalent employment shares as in Berger et al. (2025) because employers derive utility only from consumption.

The second term in the chain rule is:

$$\frac{d\ln\left(w_i/w_j\right)}{d\ln w_i} = \frac{d\left(\ln w_i - \ln w_j\right)}{d\ln w_i} = 1 - \frac{d\ln w_j}{d\ln w_i}.$$

Combining these results:

$$\begin{aligned}\frac{d\ln L_i}{d\ln w_i} &= \left[\frac{d\ln\left(L_i/L_j\right)}{d\ln\left(w_i/w_j\right)} + \frac{d\ln L_j}{d\ln\left(w_i/w_j\right)}\right]\left(1 - \frac{d\ln w_j}{d\ln w_i}\right) \\ &= \frac{d\ln\left(L_i/L_j\right)}{d\ln\left(w_i/w_j\right)}\left(1 - \frac{d\ln w_j}{d\ln w_i}\right) + \frac{d\ln L_j}{d\ln\left(w_i/w_j\right)}\left(1 - \frac{d\ln w_j}{d\ln w_i}\right).\end{aligned}$$

Finally, note that:

$$\frac{d\ln L_j}{d\ln\left(w_i/w_j\right)}\left(1 - \frac{d\ln w_j}{d\ln w_i}\right) = \frac{d\ln L_j}{d\ln\left(w_i/w_j\right)}\frac{d\ln\left(w_i/w_j\right)}{d\ln w_i} = \frac{d\ln L_j}{d\ln w_i}.$$

This gives us the final decomposition:

$$\frac{d\ln L_i}{d\ln w_i} = \frac{d\ln\left(L_i/L_j\right)}{d\ln\left(w_i/w_j\right)}\left(1 - \frac{d\ln w_j}{d\ln w_i}\right) + \frac{d\ln L_j}{d\ln w_i}.$$

Since the elasticity of substitution equals ε_b:

$$\frac{d\ln L_i}{d\ln w_i} = \varepsilon_b\left(1 - \frac{d\ln w_j}{d\ln w_i}\right) + \frac{d\ln L_j}{d\ln w_i}.$$

Structural elasticity. If we fix all the other wages in the market we get the structural labor supply elasticity:

$$\left.\frac{d\ln L_i}{d\ln w_i}\right|_{w_{-i}} = \varepsilon_b + \underbrace{\left.\frac{d\ln L_j}{d\ln w_i}\right|_{w_{-i}}}_{\text{Cross-elasticity}},$$

where we use that if all other wages are fixed $\left.\frac{d\ln w_j}{d\ln w_i}\right|_{w_{-i}} = 0$.

Now we derive the cross-elasticity fixing all the other wages. From the labor supply function:

$$L_j = s_{j|m} \times s_m \times L^S,$$

where $s_{j|m} = \frac{T_j w_j^{\varepsilon_b}}{\Phi_m}$ and $\Phi_m = \sum_{i'} T_{i'} w_{i'}^{\varepsilon_b}$.

The cross-elasticity is:

$$\left.\frac{d \ln L_j}{d \ln w_i}\right|_{w_{-i}} = \frac{d \ln s_{j|m}}{d \ln w_i} + \frac{d \ln s_m}{d \ln w_i}.$$

Since $s_{j|m} = \frac{T_j w_j^{\varepsilon_b}}{\Phi_m}$ and we hold w_j constant:

$$\frac{d \ln s_{j|m}}{d \ln w_i} = -\frac{d \ln \Phi_m}{d \ln w_i} = -\varepsilon_b s_{i|m}.$$

Since $s_m = \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi}$:

$$\frac{d \ln s_m}{d \ln w_i} = \frac{\eta}{\varepsilon_b} \frac{d \ln \Phi_m}{d \ln w_i} - \frac{d \ln \Phi}{d \ln w_i} = \eta s_{i|m} - \frac{d \ln \Phi}{d \ln w_i}.$$

In our Bertrand competition framework, where we assume unions and employers take the economy-wide aggregate Φ as given, the final term vanishes for local changes, giving us:

$$\left.\frac{d \ln L_j}{d \ln w_i}\right|_{w_{-i}} = -\varepsilon_b s_{i|m} + \eta s_{i|m} = s_{i|m}(\eta - \varepsilon_b).$$

Then the structural labor supply elasticity is:

$$\left.\frac{d \ln L_i}{d \ln w_i}\right|_{w_{-i}} = \varepsilon_b + \underbrace{\left.\frac{d \ln L_j}{d \ln w_i}\right|_{w_{-i}}}_{\text{Cross-elasticity}} = \varepsilon_b(1 - s_i) + \eta s_i,$$

which is the familiar expression we also have in the main text.

Bias. We can now compute the bias of the reduced form. The bias is:

$$\text{Bias} = \frac{d \ln L_i}{d \ln w_i} - \left.\frac{d \ln L_i}{d \ln w_i}\right|_{w_{-i}}.$$

From the decomposition:

$$\frac{d \ln L_i}{d \ln w_i} = \varepsilon_b - \varepsilon_b \frac{d \ln w_j}{d \ln w_i} + \frac{d \ln L_j}{d \ln w_i}.$$

The structural labor supply elasticity is:

$$\left.\frac{d \ln L_i}{d \ln w_i}\right|_{w_{-i}} = \varepsilon_b (1 - s_{i|m}) + \eta s_{i|m}.$$

Therefore:

$$\begin{aligned}
\text{Bias} &= \left[\varepsilon_b - \varepsilon_b \frac{d \ln w_j}{d \ln w_i} + \frac{d \ln L_j}{d \ln w_i}\right] - \left[\varepsilon_b (1 - s_{i|m}) + \eta s_{i|m}\right] \\
&= \varepsilon_b - \varepsilon_b \frac{d \ln w_j}{d \ln w_i} + \frac{d \ln L_j}{d \ln w_i} - \varepsilon_b + \varepsilon_b s_{i|m} - \eta s_{i|m} \\
&= -\varepsilon_b \frac{d \ln w_j}{d \ln w_i} + \frac{d \ln L_j}{d \ln w_i} - s_{i|m}(\eta - \varepsilon_b).
\end{aligned}$$

We know that:

$$\left.\frac{d \ln L_j}{d \ln w_i}\right|_{w_{-i}} = s_{i|m}(\eta - \varepsilon_b).$$

Substituting back and rearranging:

$$\text{Bias} = \left(\frac{d \ln L_j}{d \ln w_i} - \left.\frac{d \ln L_j}{d \ln w_i}\right|_{w_{-i}}\right) - \varepsilon_b \frac{d \ln w_j}{d \ln w_i}.$$

This is the same expression we have now in the Online Appendix.

Table S8.1: Sector Estimates

Sector Code	Sector Name	$\widehat{\beta}_b$	$\widehat{\varepsilon}_b$	$\widehat{\varphi}_b$	$\widehat{\theta}_b$	$\widehat{R}_b$
15	Food	0.76	1.53	0.23	0.13	0.11
17	Textile	0.76	1.24	0.51	0.02	0.14
18	Clothing	0.86	1.25	0.30	0.01	0.14
19	Leather	0.86	1.78	0.27	0.01	0.14
20	Wood	0.78	1.32	0.42	0.02	0.13
21	Paper	0.62	2.56	0.51	0.02	0.13
22	Printing	0.85	1.42	0.18	0.05	0.13
24	Chemical	0.67	3.02	0.08	0.14	0.16
25	Plastic	0.74	2.08	0.35	0.06	0.15
26	Other Minerals	0.66	1.46	0.43	0.05	0.15
27	Metallurgy	0.62	3.37	0.57	0.03	0.14
28	Metals	0.82	1.00	0.42	0.11	0.14
29	Machines and Equipment	0.80	1.99	0.30	0.09	0.17
30	Office Machinery	0.82	3.27	0.18	0.00	0.17
31	Electrical Equipment	0.66	2.79	0.62	0.04	0.23
32	Telecommunications	0.63	3.30	0.66	0.04	0.23
33	Optical Equipment	0.76	1.77	0.42	0.04	0.23
34	Transport	0.56	3.45	0.68	0.04	0.19
35	Other Transport	0.73	3.25	0.39	0.06	0.19
36	Furniture	0.82	1.43	0.42	0.03	0.14

Notes: Sector Code and *Sector Name* are 2-digit sector codes and names. The rest of the columns present sector estimates. $\widehat{\beta}_b$: Output elasticity of labor; $\widehat{\varepsilon}_b$: within market elasticity of substitution; $\widehat{\varphi}_b$: union bargaining power; $\widehat{\theta}_b$: intermediate good elasticities in the final good production function for 2007; $\widehat{\mathbb{R}}_b$: user cost of capital for 2007 computed following Barkai (2020).

S8 Additional estimation results

S8.1 Sector estimates

Table S8.1 presents estimates of sector b estimates of the output elasticity of labor, within-market elasticity of substitution, union bargaining power and the estimated for 2007 of the intermediate good elasticities in the final good production function and capital rental rates.

S8.2 Robustness of ε_b

Table S8.2 presents several robustness checks of the sector estimates of ε_b. The estimates from our preferred specification are in the column *Baseline* ε_b which are estimated with our firm-level instrument lagged one period. The column *2 lags* shows that the point estimates are slightly higher if we were to lag the instrument for two periods to avoid potential endogeneity to the amenity shocks.

Column $\ln(R_b k_b)$ shows that the baseline estimates are robust to including additional controls such as the logarithm of capital expenditures per worker. The estimates are overall rather similar to *Baseline* ε_b as some are above and others below our baseline estimates. The *No FE* shows that the estimated within labor market elasticities of substitutions without market-year fixed effects controlling for strategic interactions are similar to the baseline ones. Nevertheless, comparing the *No FE* estimates to the baseline ones we can clearly see that the within market elasticities of substitution are below the baseline ones for almost all the sectors. Panel A of Figure C.1 in the Online Appendix shows that under Bertrand competition, wages being strategic complements, the reduced form estimate without controlling for market changes should be below the structural parameter. That is, in the figure, the inverse of the reduced form estimate is steeper than the inverse of the structural parameter under Bertrand competition. This is supported by our estimates. Finally column $\ln(R_b k_b)$ *& no FE* shows estimated elasticities of substitution controlling for the logarithm of capital expenditures per worker but without market-year fixed effects.

S9 Additional counterfactual results

We present additional results from the counterfactuals.

S9.1 Productivity

Figure S9.1 shows productivity changes in the counterfactual with oligopsonistic competition relative to the baseline. The map shows that the biggest productivity losses happen outside big cities and some commuting zones increase overall productivity due to labor mobility across sectors.

S9.2 Perfect competition

The left Figure S9.2 presents the employment gains in the counterfactual without labor wedges across France. On the right, we have the opposite picture from the oligopsony counterfactual.

Table S8.2: Robustness of ε_b

Sector Code	Sector Name	Baseline $\widehat{\varepsilon}_b$	2 lags	$\log(R_b k_b)$	No FE	$\log(R_b k_b)$ & no FE
15	Food	1.525	1.815	1.631	1.456	1.537
17	Textile	1.235	1.504	1.058	0.913	0.584
18	Clothing	1.248	1.469	1.443	0.445	0.745
19	Leather	1.781	2.226	1.918	0.987	1.147
20	Wood	1.319	1.540	1.307	0.999	0.903
21	Paper	2.561	2.855	1.653	2.558	1.435
22	Printing	1.416	1.679	1.413	1.177	1.094
24	Chemical	3.018	3.308	2.658	2.879	2.420
25	Plastic	2.075	2.493	1.585	1.671	1.287
26	Other Minerals	1.455	1.622	1.294	1.479	1.281
27	Metallurgy	3.365	3.921	2.150	2.882	1.069
28	Metals	1.000	1.220	0.807	0.790	0.596
29	Machines and Equipment	1.990	2.375	1.639	1.769	1.394
30	Office Machinery	3.269	3.512	2.868	2.772	2.423
31	Electrical Equipment	2.790	3.340	1.906	2.329	1.418
32	Telecommunications	3.298	3.803	2.287	2.531	1.435
33	Optical Equipment	1.770	2.188	1.555	1.694	1.521
34	Transport	3.455	3.940	2.716	3.233	2.293
35	Other Transport	3.254	3.836	3.122	3.604	3.377
36	Furniture	1.428	1.706	1.387	1.129	1.019

Notes: Robustness checks of estimated ε_b. *Sector Code* and *Sector Name* are 2-digit sector codes and names. *Baseline* $\widehat{\varepsilon}_b$: baseline estimates with one lag of the instrument; *2 lags*: robustness lagging the instrument two periods; $\ln(R_b k_b)$: controls for log capital expenditure per worker in (20); *No FE*: removes the market-year fixed effect in (20); $\ln(R_b k_b)$ *& no FE*: controls for $\ln(R_b K_b)$ while removing the market-year fixed effects.

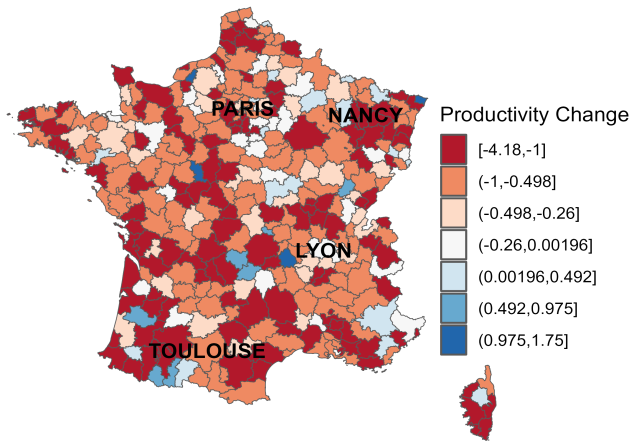
If the labor wedges were equal to one, small commuting zones would benefit most from wage and therefore employment increases.

S9.3 Alternative union objectives: Total wage bill

Here we run counterfactuals taking as as baseline union objectives total wage bills with zero outside options. Table S9.1 shows that the counterfactuals are robust to considering these alternative objectives as the results are very similar to Table 2 from the main text. Output decreases by -0.63% from removing unions and increases by 1.51% in the absence of labor wedges if we run the counterfactuals assuming that the union objectives are total wage bills in the baseline.

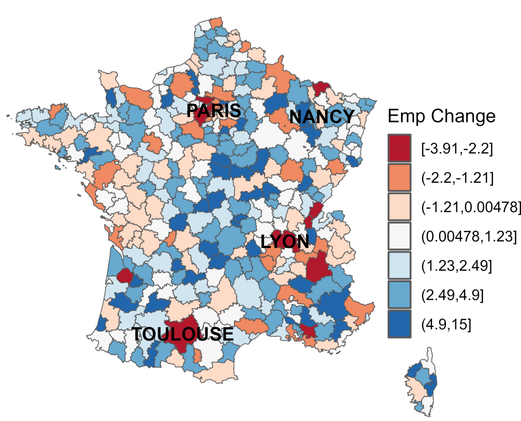
Table S9.2 shows that the workers' welfare gains and decomposition are similar to Table 3. Social welfare gains of the baseline equilibrium *Unions* are greater (3.03%) than when assuming total utility as unions' objectives (2.84%).

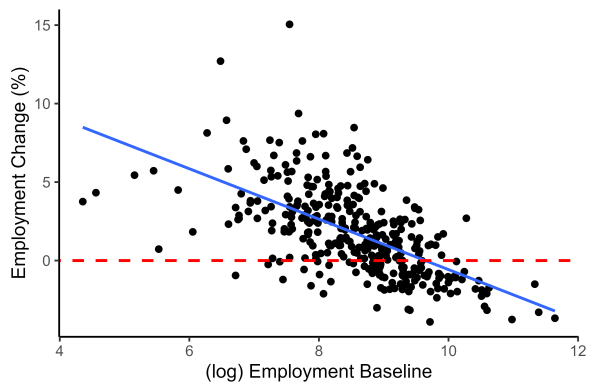
Figure S9.1: Productivity Change (%) in the Counterfactual: Oligopsonistic Competition



Notes: The map presents productivity changes with respect to the baseline economy in percentages. Each block constitutes a commuting zone. Local labor markets are aggregated up to the commuting zone. Commuting zone productivity is an employment weighted average of individual productivities. Following the discussion in Section C.4, keeping fixed the baseline revenue productivities, any change in the counterfactual comes from changes in aggregate productivities from the reallocation of workers. Counterfactuals are performed for the year 2007.

Figure S9.2: Employment Change (%) in the Counterfactual: Perfect Competition





Notes: The map presents employment changes with respect to the baseline economy in percentages within commuting zones. The counterfactual without labor wedges is performed for the year 2007. The figure in the right plots the employment change in the counterfactual versus the log of employment in the baseline. The blue line is a fitted line from an OLS regression.

Table S9.1: Wage Bill as Union Objective: Output and Welfare

	Labor share (%)	Gains (%)		
		ΔY	Δ Wage	Δ Welfare (L)
Baseline $\lambda(\mu, \varphi_b)$	50.38	-	-	-
Counterfactuals				
Oligopsony $\lambda(\mu, 0) = \mu_i$	36.69	-0.63	-27.64	-30.13
No wedges $\lambda(1, 0) = 1$	73.32	1.51	47.73	44.50
Monopsony $\lambda(\mu, 0) = \frac{\varepsilon_b}{\varepsilon_b+1}$	47.79	1.72	-3.51	-7.30

Notes: Results in percentages. *Labor share*: aggregate labor share. The last three columns are changes relative to the baseline. ΔY: aggregate output, Δ *Wage*: aggregate wage (employment weighted average). Δ *Welfare (L)*: median expected welfare of the workers. *Oligopsony*: counterfactual without unions $\lambda_i = \mu_i$; *No wedges*: wedge equal to one (perfect competition); *Monopsony*: monopsonistic competition (infinitesimal firms) without unions.

Table S9.2: Wage Bill as Union Objective: Welfare Decomposition

	$\Delta\mathcal{W}_c$ (%)	Share $\Delta\mathcal{W}_c$		$\Delta\mathcal{S}_c$ (%)	Share $\Delta\mathcal{S}_c$	
		Reallocation	Rent-sharing		AE	RE
Unions	43.12	73.31	26.69	3.03	78.51	21.49
No wedges $\lambda(1, 0) = 1$	106.8	51.15	48.85	3.44	81.28	18.72
Monopsony $\lambda(\mu, 0) = \frac{\varepsilon_b}{\varepsilon_b+1}$	32.67	78.64	21.36	2.03	103.73	-3.73

Notes: Oligopsony as reference. $\Delta\mathcal{W}_c$: workers' welfare gains; *Reallocation* and *Rent-sharing* share of welfare gains in (22). $\Delta\mathcal{S}_c$: social welfare gains; *AE* and *RE* share of $\Delta\mathcal{S}_c$ from efficiency and redistribution in (23). *Unions*: baseline with bargaining; *No wedges*: wedges equal to one; *Monopsony*: infinitesimal firms without unions.

S9.4 Increasing union bargaining power

Table S9.3 presents the counterfactual labor share and gains of output, wages and welfare as we increase the union bargaining powers. The aggregate output gains in Table S9.3 resume the sector level output gains of Figure D.1a of the Online Appendix.

Increasing the union bargaining powers $\varphi_b^{1-\kappa}$ by bringing κ closer to one increases output gains. When $\kappa = 1$, the union bargaining powers are set to one and all the sectors reach the effective maximum bargaining power φ_b^* where $\lambda_i = \psi_i = 1\, \forall i \in \mathcal{I}_b$. This equilibrium is therefore equivalent to the perfect competition allocation with *No wedges* in Table 2. Inspecting Table S9.3 we see that increasing the union bargaining powers increases output, wages and workers' welfare as higher union bargaining power countervails the negative effects of employer labor market power along the labor supply.

Table S9.3: Counterfactuals: Increasing Union Bargaining Power

	Labor share (%)	Gains (%)		
		ΔY	Δ Wage	Δ Welfare (L)
Baseline	50.38	-	-	-
Counterfactuals				
$\kappa = 0.1$	51.76	0.10	2.84	2.87
$\kappa = 0.2$	53.31	0.20	6.03	6.05
$\kappa = 0.3$	55.05	0.33	9.63	9.61
$\kappa = 0.4$	57.01	0.47	13.69	13.57
$\kappa = 0.5$	59.21	0.63	18.28	17.98
$\kappa = 0.6$	61.71	0.80	23.47	22.89
$\kappa = 0.7$	64.52	0.99	29.35	28.34
$\kappa = 0.8$	67.69	1.20	35.99	34.34
$\kappa = 0.9$	71.22	1.43	43.40	40.85
$\kappa = 1$	73.33	1.65	47.96	44.32

Notes: Results in percentages. *Labor share*: aggregate labor share. The last three columns are changes relative to the baseline. ΔY: aggregate output, Δ *Wage*: aggregate wage (employment weighted average). Δ *Welfare (L)*: median expected welfare of the workers. *Baseline*: Baseline equilibrium; *Counterfactuals* where we increase union bargaining powers as $\varphi_b^{1-\kappa}$ with utilitarian union objectives. Bargaining powers increasing in κ and equal to one when $\kappa = 1$.

S9.5 Misaligned objectives

Table S9.4 replicates the counterfactual of increasing union bargaining powers as κ tends to one but with misaligned objectives on insiders. When κ is below 0.5, increasing union bargaining powers induce movements along the labor supply without leading to equilibrium unemployment. When $\kappa > 0.5$, high union bargaining powers generate some unemployment but overall we find that output would be higher than in the baseline except for the limit case when $\kappa = 1$. Expected wages and workers' welfare are higher than in the baseline for all the values of κ but the expected wage and welfare gains peak at $\kappa = 0.9$. When unions have all the bargaining power, $\kappa = 1$, the unemployment rates spike to 11.52% and output is 7% lower than in the baseline. These negative effects induce that the expected wage and workers' welfare gains are lower with full bargaining power than when $\kappa = 0.9$.

S9.6 Sector wage floors

Figure S9.3 shows the distribution of employers per regions with sector wage floors set at percentile 1 of the observed wages. Wage floors being low, most firms are unconstrained in Region

Table S9.4: Counterfactuals: Misaligned Objectives

	LS (%)	Unemp. Rate (%)	Gains (%)		
			ΔY	Δ Exp. Wage	Δ Welfare (L)
Baseline	50.38	0.00	-	-	-
Counterfactuals					
$\kappa = 0.1$	51.76	0.00	0.10	2.84	2.87
$\kappa = 0.2$	53.31	0.00	0.20	6.03	6.05
$\kappa = 0.3$	55.05	0.00	0.33	9.63	9.61
$\kappa = 0.4$	57.01	0.00	0.47	13.69	13.57
$\kappa = 0.5$	59.21	0.00	0.63	18.28	17.98
$\kappa = 0.6$	61.69	0.14	0.73	23.34	22.67
$\kappa = 0.7$	64.48	0.35	0.78	29.01	27.76
$\kappa = 0.8$	67.66	0.58	0.85	35.45	33.39
$\kappa = 0.9$	71.18	1.79	0.27	41.68	37.61
$\kappa = 1$	73.33	11.52	-7.00	35.37	23.78

Notes: Results in percentages. *Labor share*: aggregate labor share; *Unemp. Rate*: aggregate unemployment rate. The last three columns are changes relative to the baseline. ΔY: aggregate output, Δ *Exp. Wage*: aggregate expected wage (labor supply weighted average). Δ *Welfare (L)*: median expected welfare of the workers. *Baseline*: Baseline equilibrium; *Counterfactuals* where we increase union bargaining powers as $\varphi_b^{1-\kappa}$ with misaligned union objectives. Bargaining powers increasing in κ and equal to one when $\kappa = 1$.

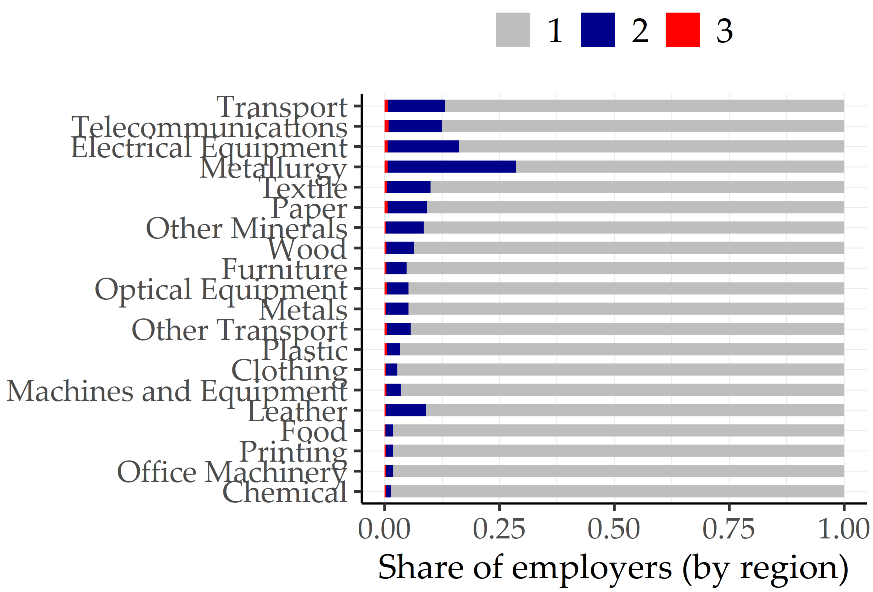
I as shown by the gray bars. Some firms would be constrained by the sector wage floors if they were operating in oligopsony (blue bars) in the absence of sector minimum wages. A handful of firms per sector would be in Region III where they would be leaving a fraction of their labor supply slack. For low sector wage floors, the vast majority of firms would therefore be on regions where, either sector minimum wages are irrelevant (Region I) or may bring wages closer to perfect competition (Region II).

S9.7 Employer exit

We consider another counterfactual scenario where increasing the bargaining power might lead to exit of some employers as they need to cover fixed operating costs. To abstract from complications of entry games with strategic interactions, we assume the baseline equilibrium as one where the most profitable potential entrants survive, and focus instead on the potential exit of those employers when the bargaining powers increase.

We assume the employers must pay an operation fixed cost f before the bargaining stage.

Figure S9.3: Sector Wage Floors: Share of Firms per Region



Notes: Counterfactual with sector wage floor at the 1st percentile of baseline wages. Share of employers unconstrained in Region I (gray), constrained by the wage floor if they were competing oligopsonistically in Region II (blue) and constrained and rationing in Region III (red).

This implies that the fixed cost is sunk and employers' outside option remains zero.[4] For any employer i to remain operating in equilibrium we need $\Pi_i \geq f$. We calibrate $f = \min_{i \in \mathcal{I}} \Pi_i$, so that the least profitable employer in the baseline makes zero profits once the fixed cost is taken into account. This calibration gives us the maximum amount of exit in a counterfactual, allowing us to bound the effects of exit.

By increasing the union bargaining powers, some employers become unprofitable and exit. Table S9.5 summarizes the results of the counterfactual with exit. The output and welfare gains remain almost identical to the baseline (Table S9.3) as less than 0.1% of firms exit when all the union bargaining powers are set to one.

S9.8 Robustness: Local labor market definition

We present main counterfactuals with alternative definitions of the local labor market. Table S9.6 defines the local labor market as commuting zone and 3-digit industry combinations while

[4]This is akin to the specification of Kim and Vogel (2021), where workers and firms bargain after the firm pays a vacancy cost which is sunk at that stage.

Table S9.5: Counterfactuals: Firm Exit

	Number of i	Labor share (%)	Gains (%) ΔY	Δ Wage	Δ Welfare (L)
Baseline	278,091	50.38	-	-	-
Counterfactuals					
$\kappa = 0.1$	278,091	51.76	0.10	2.84	2.87
$\kappa = 0.2$	278,091	53.31	0.20	6.03	6.05
$\kappa = 0.3$	278,091	55.05	0.33	9.63	9.61
$\kappa = 0.4$	278,089	57.01	0.47	13.69	13.57
$\kappa = 0.5$	278,082	59.21	0.63	18.28	17.98
$\kappa = 0.6$	278,079	61.71	0.80	23.47	22.89
$\kappa = 0.7$	278,073	64.52	0.99	29.35	28.34
$\kappa = 0.8$	278,049	67.69	1.20	35.99	34.34
$\kappa = 0.9$	277,945	71.22	1.43	43.40	40.85
$\kappa = 1$	277,817	73.33	1.65	47.96	44.32

Notes: Number of i: number of active union-employer pairs; *Labor share*: aggregate labor share. The last three columns are changes relative to the baseline. ΔY: aggregate output, Δ *Wage*: aggregate wage (employment weighted average). Δ *Welfare (L)*: median expected welfare of the workers. *Baseline*: Baseline equilibrium; *Counterfactuals* where we increase union bargaining powers as $\varphi_b^{1-\kappa}$ with utilitarian union objectives and allowing for employer exit. Bargaining powers increasing in κ and equal to one when $\kappa = 1$.

Table S9.7 presents robustness to defining the local labor market as commuting zone times 2-digit occupations.

S10 Data details

In this section we provide details about sample selection and variable construction.

S10.1 Sample selection

Ficus/Fare. This data source comes from tax records therefore we observe yearly firm information. We exclude the source tables belonging to public firms.[5] Before 2000 we take table sources in euros and from 2001 onward we use data from consolidated economic units.[6] After excluding firms without a firm identifier, the raw data sample contains about 29 million

[5]We only use the Financial units (*FIN*) and Other units (*TAB*) tables and exclude Public administration (*APU*).

[6]The profiling of big groups consolidates legal units into economic units. In 2001 the Peugeot-Citroën PSA was treated, Renault in 2003 and the group Accor in 2005. This implies the definition of new economic entities and would therefore lead to the creation of new firm identifiers. Given the potential impact of big establishments in local labor markets we opted to maintain them.

Table S9.6: Robustness: Commuting Zone $\times$ Sector

	Labor share (%)	Gains (%)		
		ΔY	Δ Wage	Δ Welfare (L)
Baseline $\lambda(\mu, \varphi_b)$	50.47	-	-	-
Counterfactuals				
Oligopsony $\lambda(\mu, 0) = \mu_i$	37.25	-0.63	-26.67	-28.92
No wedges $\lambda(1, 0) = 1$	73.32	1.69	47.73	44.35

Notes: Results in percentages. *Labor share*: aggregate labor share. The last three columns are changes relative to the baseline. ΔY: aggregate output, Δ *Wage*: aggregate wage (employment weighted average). Δ *Welfare (L)*: median expected welfare of the workers. *Oligopsony*: counterfactual without unions $\lambda_i = \mu_i$; *No wedges*: wedge equal to one (perfect competition); *Monopsony*: monopsonistic competition (infinitesimal firms) without unions.

Table S9.7: Robustness: Commuting Zone $\times$ 2-digit Occupation

	Labor share (%)	Gains (%)		
		ΔY	Δ Wage	Δ Welfare (L)
Baseline $\lambda(\mu, \varphi_b)$	50.46	-	-	-
Counterfactuals				
Oligopsony $\lambda(\mu, 0) = \mu_i$	44.09	-0.13	-12.73	-12.64
No wedges $\lambda(1, 0) = 1$	72.90	0.34	44.97	44.10

Notes: Results in percentages. *Labor share*: aggregate labor share. The last three columns are changes relative to the baseline. ΔY: aggregate output, Δ *Wage*: aggregate wage (employment weighted average). Δ *Welfare (L)*: median expected welfare of the workers. *Oligopsony*: counterfactual without unions $\lambda_i = \mu_i$; *No wedges*: wedge equal to one (perfect competition); *Monopsony*: monopsonistic competition (infinitesimal firms) without unions.

firms, of which about 2.8 million are manufacturing firms.[7] Manufacturing sector (sector code equal to *D*) constitutes on average 10% of the observations, 19.2% of value added and 27.2% of employment.

Postes. *DADS Postes* covers all the employment spells of a salaried employee per year. If a worker has several spells in a year we would have multiple observations. The main benefit of this employer-employee data source is that we can know the establishment and employment location of the workers. We exclude workers in establishments with fictitious identifiers (SIREN starting by F) and in public firms. For every establishment identifier (SIRET) we sum the wage bill and the full time equivalent number of employees.

[7] We consider a missing firm identifier (SIREN) also if the identifier equals to zero for all the 9 digits.

Merged data. After merging both data sources, we end up with data that include yearly establishment observations. After the filters and merging the sample consists of 1.3 million firms and 1.6 million establishment observations. In the process of filtering and merging, about half of the original firms are lost. Wages and value added are deflated using the Consumer Price Index.[8]

Labor and wage data, coming from the balance sheets (at the firm level) and the one from employee records, needs to be consolidated. In order to be consistent with balance sheet information we assign labor and employment coming from *FICUS* to the establishments according to their respective shares. We proceed in several steps. First, we filter out observations with no wage or employment information from *Postes* from firms present at different commuting zones. Second, we get rid of observations with no labor, capital and wage bill information coming from *FICUS* and also observations with non existing or missing commuting zone. Third, we aggregate employee data to the firm times commuting zone level.[9] What we call employer throughout the text is the entity aggregated at the commuting zone times occupation level. Then we compute the labor and wage shares of these entities out of the firm's aggregates. Finally, we split firm data from the balance sheet according to those shares. This procedure leaves the firms in a unique commuting zone and occupation with their balance sheet data at the employer but allows to split wage bill and employment data coming from the balance sheet for firms with multiple employers. Employer wage is simply the average wage. That is, wage bill over total full time equivalent employees.

We further exclude Tobacco (2-digits industry code 16), Refineries & Nuclear industry (code 23) and Recycling (code 37). We finally get rid of the outliers reducing the sample 1.5% and finish with 4,206,408 establishment-occupation-year observations that belong to 1.25 million firms.[10]

[8] Nominal variables are expressed in constant 2015 euros.

[9] Data from 1994 and 1995 do not have commuting zone information. We therefore impute it using correspondence tables between city code and commuting zone. A city code has 5 digits coming from the department and city. Some commuting zone codes beyond the 2 missing years were modified or cleaned. City codes (*commune* codes) of Paris, Marseille and Lyon were divided into different *arrondissements*. We assign them codes 75056, 13055 and 69123 respectively. Then we proceed to the cleaning of commuting zones by assigning to the non existing codes the one corresponding to the city where the establishment is located. We get rid of non matched or missing commuting zone codes. We aggregate the data coming from *Postes* at the commuting zone times occupation level after this cleaning.

[10] We get rid of outliers by truncating the sample at the 0.5% below and 99.5% of the wage distribution. We furthermore remove the outliers of revenue per worker by trimming the 0.5% below.

S10.2 Variable construction

Ficus:

- Value added: value added net of taxes (*VACBF*). We restrict to firms with strictly positive value added.[11]

- Capital: tangible and intangible capital without counting depreciation. It is the sum of the variables *IMMOCOR* and *IMMOINC*.

- Employment: full time equivalent employment at the firm (*EFFSALM*).

- Wage bill: gross total wage bills. Is the sum of wages (*SALTRAI*) and firm taxed (*CHARSOC*).[12]

- Industry: industry classification comes from *APE*. The sub-industries h are 3 digit industries and sectors b are at two digits.

Postes:

- Occupation: original occupation categories come from the two digit occupations (*CS2*). We group occupations with first digits 2 and 3 into a unique occupation group.[13] This regrouping is done to avoid substantial changes in occupation groups between 1994 and 2007. Observations with missing occupation information are excluded.

- Employment: full time equivalent at the establishment-occupation level (*etp*).

- Wage: is the gross wage (per year) of individual worker (*sbrut*). If the spell is less than a year is the gross wage corresponding to the spell.

- Commuting zone: depending on the year, the commuting zone classification is taken from the variable *zemp* or *zempt*. Commuting zone information is missing for the years 1994 and 1995 and is imputed using the city codes.[14]

[11] We follow the advise of the French statistical instiute (INSEEE) in using net value added to perform comparisons across industries.

[12] For firms declaring at the BIC-BRN regime (*TYPIMPO*= 1) we only take *SALTRAI* .

[13] Occupations with first digit 1 and 7 are excluded. They constituted less than 0.05% of the matched sample.

[14] City codes are the concatenation of department (*DEP*) and city (*COM*).

S10.3 Construction of required rates

In order to construct the required rates for the different sectors we follow the methodology proposed by Barkai (2020) using the Capital Input Data from the EU KLEMS database, December 2016 revision. In this dataset one can find, for a given industry, different depreciation rates and price indices for different types of capital. The types of capital that are present in the manufacturing sector are: Computing Equipment, Communications Equipment, Computer Software and Databases, Transport Equipment, Buildings and structures (non-residential), and Research and Development. We construct a required rate for each of the industries at the 2 digit level defined in the NAF classification. However, unlike the NAF classification, that we have up to 20 different industries, there are only 11 industries classified within manufacturing within the EU KLEMS database. Any industry classification in EU KLEMS is just an aggregation of the 2 digit industry classification in NAF. Therefore, there are industries within the NAF classification that will have the same required rate of return on capital.

For a type of capital s and sector b, we define the the required rate of return R_{sb} as:

$$R_{sb} = \left(i^D - \mathbb{E}\left[\pi_{sb}\right] + \delta_{sb}\right),$$

where i^D is a the cost fo debt borrowing in financial markets, and π_{sb} and δ_{sb} are, respectively, the inflation and depreciation rates of capital type s in sector b.

Then we define the total expenditures on capital type s in sector b as:

$$E_{sb} = R_{sb}P^K_{sb}K_{sb},$$

where $P^K_{sb}K_{sb}$ is the nominal value of capital stock of type s. Summing over all types of capital within a sector we can obtain the total expenditures of capital of such sector:

$$E_b = \sum_{sb} R_{sb}P^K_{sb}K_{sb}.$$

Table S11.1: Establishment-occupation Summary Statistics

Variable	Mean	P25	Median	P75
L_i	11.03	1.06	2.27	6.22
$w_i L_i$	365.32	31.58	72.12	197.90
w_i	33.90	20.86	27.47	39.56
$s_{i\|m}$	0.20	0.01	0.05	0.23

Notes: The number of i in the whole sample is 4,206,408. L_i: full time equivalent employment at the establishment-occupation i, $w_i L_i$: wage bill at i, w_i: establishment-occupation wage or wage per FTE, $s_{i|m}$: employment share out of the local labor market. All the nominal variables are in thousands of constant 2015 euros.

Multiplying and dividing by the total nominal value of capital stock we obtain:

$$\sum_s R_{sb} P^K_{sb} K_{sb} = \underbrace{\sum_s \frac{P^K_{sb} K_{sb}}{\sum_{s'} P^K_{s'b} K_{s'b}} R_{sb}}_{R_b} \underbrace{\sum_s P^K_{sb} K_{sb}}_{P^{Kb} K_b},$$

where the first term R_b is the interest rate that we use in the model.

S10.4 Other sources

The source to construct commuting zones from city codes is https://www.insee.fr/fr/information/2114596 and the CPI data comes from https://www.insee.fr/fr/statistiques/serie/001643154.

S11 Summary statistics

Tables S11.1, S11.2, S11.3 and S11.4 contain respectively summary statistics of establishment-occupations, 3-digit industries, local labor markets and commuting zones for the year 2007, which is the year we use for our counterfactuals. Table S11.5 presents worker transition probabilities across occupations, industries and commuting zones.

Table S11.2: Sub-industry Summary Statistics. Baseline Year

Variable	Mean	P25	Median	P75
N_h	2,866.94	504.00	1,274.00	2,700.00
L_h	30,644.61	7,587.00	15,728.00	50,229.00
$\widehat{w}_h$	34.67	29.67	33.01	37.55
LS_h	0.52	0.48	0.52	0.58
KS_h	0.26	0.17	0.23	0.31

Notes: There are 97 3-digit industries, or sub-industries, in the sample. N_h: number of i per 3-digit industry h, L_h: total employment of h, $\widehat{w}_h$: average establishment wage of h, LS_h: labor share, and KS_h: capital share. We get the capital shares following Barkai (2020). All the nominal variables are in thousands of constant 2015 euros.

Table S11.3: Local Labor Market Summary Statistics. Baseline Year

Variable	Mean	P25	Median	P75
N_m	4.77	1.00	2.00	4.00
L_m	51.04	2.80	9.46	35.14
$\widehat{w}_m$	36.59	24.27	30.16	42.38
$\overline{w}_m$	36.42	24.10	30.04	42.27
HHI($s_{i\|m}$)	0.67	0.38	0.68	1.00
HHI($s^w_{i\|m}$)	0.67	0.39	0.69	1.00

Notes: There are 57,940 local labor markets in 2007. N_m: number of competitors in the local labor market m, L_m: total employment in m, $\widehat{w}_m$: mean w_i in m, $\overline{w}_m$: weighted average wage at m with employment shares as weights, HHI($s_{i|m}$) and HHI($s^w_{i|m}$) are respectively the Herfindahls with employment and wage shares. All the nominal variables are in thousands of constant 2015 euros.

Table S11.4: Commuting Zones Summary Statistics. Baseline Year

Variable	Mean	P25	Median	P75
N_n	781.14	267.75	461.50	867.00
L_n	8,349.22	2,547.65	5,317.90	10,280.56
$\overline{L}_n$	11.36	8.10	10.97	13.62
$\widehat{w}_n$	34.45	32.92	34.38	35.76

Notes: There are 356 commuting zones in the sample. N_n: number of establishment-occupations i at the CZ, L_n: full time equivalent employment at CZ, $\overline{L}_n$: average L_i at n, $\widehat{w}_n$: mean w_i at n in thousands of constant 2015 euros.

Table S11.5: Transition Probabilities

Occupation	CZ	Industry	Trans. Prob. FTE (%)	Trans. Prob. (%)
0	0	0	91.388	91.012
0	0	1	2.368	2.357
0	1	0	0.019	0.018
1	0	0	6.028	6.400
1	0	1	0.196	0.211
1	1	0	0.001	0.001
1	1	1	0.000	0.000

Notes: The transition rates are computed over the whole sample period 1994-2007. *Occupation*: indicator of occupational change, *CZ*: indicator of commuting zone change, *Industry*: indicator of 3-digit industry change, *Trans. Prob. FTE*: unconditional transition probabilities based on full time equivalent units, *Trans. Prob.*: unconditional transition probabilities based on counts of working spells independently of duration and part-time status.

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