

## ONLINE APPENDIX

# Union and Firm Labor Market Power

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This Appendix is organized as follows. Section A shows the proofs of the propositions in the main text. Section B presents the model derivations for the baseline equilibrium. Section C presents the details of our identification strategy and additional estimation results. Section D provides additional details about the counterfactual exercises. Section E provides details on our reduced form exercise and the theoretical link of the reduced-form with our model.

## A Proofs

**Proof of Proposition 1** Abstracting from wage bargaining, the establishment problem is:

$$\max_{w_{io}, K_{io}} P_b \sum_{o=1}^O \tilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b} - \sum_{o=1}^O w_{io} L_{io}(w_{io}) - R_b \sum_{o=1}^O K_{io},$$

The first order condition with respect to the wage is:

$$P_b \frac{\partial F}{\partial L_{io}} \frac{\partial L_{io}}{\partial w_{io}} = L_{io}(w_{io}) + w_{io} \frac{\partial L_{io}}{\partial w_{io}},$$

where the derivative of the labor supply  $L_{io}$  with respect to  $w_{io}$  is:

$$\begin{aligned} \frac{\partial L_{io}}{\partial w_{io}} &= \frac{L \Gamma_b^\eta}{\Phi} \left( \left[ \frac{\varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1} \Phi_m - T_{io} w_{io}^{\varepsilon_b} \varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1}}{\Phi_m^2} \right] \Phi_m^{\eta/\varepsilon_b} + \eta \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} \Phi_m^{\eta/\varepsilon_b - 1} T_{io} w_{io}^{\varepsilon_b - 1} \right) \\ &= \varepsilon_b \frac{L_{io}}{w_{io}} - \varepsilon_b \frac{L_{io}}{w_{io}} \frac{L_{io}}{L_m} + \eta \frac{L_{io}}{w_{io}} \frac{L_{io}}{L_m} = \frac{L_{io}}{w_{io}} \left( \varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right). \end{aligned}$$

Substituting this last derivative into the first order condition we get:

$$\begin{aligned} L_{io} + L_{io} \left( \varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right) &= P_b \frac{\partial F}{\partial L_{io}} \frac{L_{io}}{w_{io}} \left( \varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right) \\ \Rightarrow w_{io} &= \frac{\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m}}{\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} + 1} P_b \frac{\partial F}{\partial L_{io}} = \mu(s_{io|m}) P_b \frac{\partial F}{\partial L_{io}}. \quad \square \end{aligned}$$

**Proof of Proposition 2.** Substituting (7) into (9) and using  $\frac{\beta_b}{1-\alpha_b} = 1 - \delta \in [0, 1]$ :

$$w_{io} = \left( \lambda(\mu_{io}, \varphi_b) \beta_b \frac{A_{io}}{(T_{io} \Gamma_b^\eta)^\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} \Phi_m^{\frac{\delta(1-\eta/\varepsilon_b)}{1+\varepsilon_b \delta}} P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b \delta)}} \left( \frac{\Phi}{L} \right)^{\frac{\delta}{1+\varepsilon_b \delta}}. \quad (\text{A1})$$

From (7), the equilibrium employment share of the establishment-occupation is:

$$s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\sum_{j \in \mathcal{I}_m} T_{jo} w_{jo}^{\varepsilon_b}} = \frac{T_{io}^{\frac{1}{1+\varepsilon_b \delta}} \lambda_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}} A_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}{\sum_{j \in \mathcal{I}_m} T_{jo}^{\frac{1}{1+\varepsilon_b \delta}} \lambda_{jo}^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}} A_{jo}^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}},$$

where we used equation (A1) in the second step and simplified terms. The solutions of the labor wedge  $\lambda_{io}(\mu_{io}, \varphi_b)$  and the markdown come respectively from equations (9) and (8).  $\square$

**Proof of Proposition 3. Existence.** We follow closely the proof by Kucheryavyy (2012). Define the right hand side of (A1) as:  $f_{io}(\mathbf{w}) = [\lambda(\mu(s(\mathbf{w})))]^{\frac{1}{1+\varepsilon_b \delta}} c_{io}$ , where  $\mathbf{w}$  denotes the vector formed by  $\{w_{io}\}$ , we simplified the notation of the wedge  $\lambda(\mu_{io}, \varphi_b)$  from the main text by getting rid of the second argument.  $c_{io} = \left( \beta_b \frac{A_{io}}{(T_{io} \Gamma_b^\eta)^\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} \Phi_m^{\frac{(1-\eta/\varepsilon_b)\delta}{1+\varepsilon_b \delta}} P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b \delta)}} \left( \frac{\Phi}{L} \right)^{\frac{\delta}{1+\varepsilon_b \delta}}$  is an establishment-occupation specific parameter. We consider  $\Phi_m$  and  $\Phi$  as constants.

Under the assumption  $0 < \eta < \varepsilon_b$ , the function  $\mu(s) = \frac{\varepsilon_b(1-s) + \eta s}{\varepsilon_b(1-s) + \eta s + 1}$  is decreasing in  $s$ , the employment share out of the local labor market. Therefore, we can conclude that the wedge  $\lambda(\mu(s)) = (1 - \varphi_b)\mu(s) + \varphi_b \frac{1}{1-\delta}$  is also decreasing in  $s$ . The employment share has bounds  $0 \leq s \leq 1$ , which implies  $(1 - \varphi_b) \frac{\eta}{\eta+1} + \varphi_b \frac{1}{1-\delta} \leq \lambda(\mu(s)) \leq (1 - \varphi_b) \frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b \frac{1}{1-\delta}$ . Also,  $1 + \varepsilon_b \delta > 0$ . Therefore, it follows that  $f_{io}(\mathbf{w})$  is bounded:

$$\left( (1 - \varphi_b) \frac{\eta}{\eta+1} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} c_{io} \leq f_{io}(\mathbf{w}) \leq \left( (1 - \varphi_b) \frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} c_{io}.$$

If the number of competitors in market  $m$  is  $N_m > 0$ , we can define the compact set  $S$  where  $f_{io}(\mathbf{w})$  maps into itself as:

$$S = \left[ \left( (1 - \varphi_b) \frac{\eta}{\eta+1} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} c_1, \left( (1 - \varphi_b) \frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} c_1 \right] \times \dots \\ \times \left[ \left( (1 - \varphi_b) \frac{\eta}{\eta+1} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} c_{N_m}, \left( (1 - \varphi_b) \frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\varepsilon_b \delta}} c_{N_m} \right].$$

The function  $f_{io}(\mathbf{w})$  is continuous in wages on  $S$ . We can therefore apply Brouwer's fixed point theorem and claim that at least one solution exists.  $\square$

**Uniqueness.** In the Supplemental Material we present a Theorem and a Corollary from [Allen et al. \(2016\)](#) that we use to establish uniqueness. Define  $g : \mathbb{R}_{++}^n \rightarrow \mathbb{R}^n$  for some  $n \in \{1, \dots, N\}$ :

$$g_{io}(\mathbf{w}) = f_{io}(\mathbf{w}) - w_{io}, \quad \forall i \in \{1, \dots, N_m\}.$$

We want to prove that the solution satisfying  $g(\mathbf{w}) = 0$  is unique. In order to do so, we first need to show that  $g(\mathbf{w})$  satisfies the gross substitution property ( $\frac{\partial g_{io}}{\partial w_{jo}} > 0$  for any  $j \neq i$ ).

Taking the partial derivative of  $g_{io}$  with respect to  $w_{jo}$  for any  $j \neq i$ :

$$\frac{\partial g_{io}}{\partial w_{jo}} = \frac{\partial f_{io}(\mathbf{w})}{\partial \lambda(\mu(s(\mathbf{w})))} \times \frac{\partial \lambda(\mu(s_{io|m}))}{\partial \mu(s_{io|m})} \times \frac{\partial \mu(s_{io|m})}{\partial s_{io|m}} \times \frac{\partial s_{io|m}}{\partial w_{jo}},$$

where  $\frac{\partial f_{io}(\mathbf{w})}{\partial \lambda(\mu(s(\mathbf{w})))} = \frac{1}{1+\varepsilon_b \delta} \frac{f_{io}(\mathbf{w})}{\lambda(\mu(s(\mathbf{w})))} > 0$ . We have that  $\frac{\partial \lambda(\mu(s_{io|m}))}{\partial \mu(s_{io|m})} > 0$ . We previously established that  $\frac{\partial \mu(s_{io|m})}{\partial s_{io|m}} < 0$  under the assumption that  $0 < \eta < \varepsilon_b$ . The share of an establishment  $i$  with occupation  $o$  in sub-market  $m$  is:  $s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\sum_{j \in \mathcal{I}_m} T_{jo} w_{jo}^{\varepsilon_b}}$ . Clearly,  $\frac{\partial s_{io|m}}{\partial w_{jo}} < 0$  for any  $i \neq j$ . Therefore  $\frac{\partial g_{io}}{\partial w_{jo}} > 0$  for any  $i \neq j$  and  $g$  satisfies the gross-substitution property required by Theorem 1 in the Supplemental Material.

The remaining condition to prove to use Corollary 1 in the Supplemental Material is simply that  $f_{io}(\mathbf{w})$  is homogeneous of a degree smaller than 1.<sup>1</sup> Clearly,  $f_{io}(\mathbf{w})$  is homogeneous of degree 0 like the markdown function  $\mu(s_{io|m})$ . Therefore, the function  $g$  satisfies the conditions of Corollary 1, and we conclude that there exists at most one solution satisfying  $g(\mathbf{w}) = 0$ .  $\square$

**Proof of Proposition 4.** Aggregating establishment-occupation output (6) and using the restriction  $\frac{\beta_b}{1-\alpha_b} = 1 - \delta \in [0, 1]$ , the local labor market output is:

$$Y_m = \sum_{i \in \mathcal{I}_m} y_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} L_{io}^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} s_{io|m}^{1-\delta} L_m^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_m A_m L_m^{1-\delta},$$

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<sup>1</sup>The degree of homogeneity of  $h_{io}(\mathbf{w}) = w_{io}$  is 1.

where the local labor market productivity and misallocation are measured as:

$$\Omega_m \equiv \sum_{i \in \mathcal{I}_m} \frac{A_{io}}{A_m} s_{io|m}^{1-\delta} \quad A_m \equiv \sum_{i \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \quad \tilde{s}_{io|m} = \frac{\left(T_{io}^{1/\varepsilon_b} A_{io}\right)^{\varepsilon_b/1+\varepsilon_b\delta}}{\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\varepsilon_b/1+\varepsilon_b\delta}}.$$

$\tilde{s}_{io|m}$  comes from Proposition 2 with constant  $\lambda$ . Aggregating to sector level using (4):

$$Y_b = \sum_{m \in \mathcal{M}_b} Y_m = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{m \in \mathcal{M}_b} \Omega_m A_m L_m^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_b A_b L_b^{1-\delta}. \quad (\text{A2})$$

The sector level measures of productivity and misallocation are:

$$\begin{aligned} \Omega_b &\equiv \sum_{m \in \mathcal{M}_b} \Omega_m A_m / A_b s_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} A_{io} / A_b s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}, \\ A_b &\equiv \sum_{m \in \mathcal{M}_b} A_m \tilde{s}_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta}, \\ \tilde{s}_{m|b} &= \frac{\left[\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\varepsilon_b/1+\varepsilon_b\delta}\right]^{\eta(1+\varepsilon_b\delta)/\varepsilon_b(1+\eta)}}{\sum_{m' \in \mathcal{M}_b} \left[\sum_{j' \in \mathcal{I}_{m'}} \left(T_{j'o}^{1/\varepsilon_b} A_{j'o}\right)^{\varepsilon_b/1+\varepsilon_b\delta}\right]^{\eta(1+\varepsilon_b\delta)/\varepsilon_b(1+\eta)}}. \end{aligned}$$

From (9), the establishment wage bill is:  $w_{io} L_{io} = \beta_b \lambda_{io} P_b y_{io}$ , and aggregating to  $m$ :

$$\begin{aligned} \sum_{i \in \mathcal{I}_m} w_{io} L_{io} &= \beta_b \sum_{i \in \mathcal{I}_m} \lambda_{io} P_b y_{io} = \beta_b \sum_{i \in \mathcal{I}_m} \lambda_{io} \frac{P_b y_{io}}{P_b Y_m} P_b Y_m = \beta_b \lambda_m P_b Y_m, \\ \lambda_m &\equiv \sum_{i \in \mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_m A_m} s_{io|m}^{1-\delta} \end{aligned}$$

where  $\lambda_m$  is a value added weighted sum of  $\lambda_{io}$ . Aggregating to the sector,

$$\begin{aligned} \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} w_{io} L_{io} &= \beta_b \sum_{m \in \mathcal{M}_b} \lambda_m \frac{P_b Y_m}{P_b Y_b} P_b Y_b = \beta_b \lambda_b P_b Y_b, \\ \lambda_b &\equiv \sum_{m \in \mathcal{M}_b} \lambda_m \frac{A_m \Omega_m}{\Omega_b A_b} s_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_b A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}. \quad \square \end{aligned}$$

**Proof of Proposition 5.** Equation (A1) can be separated into two terms. First, a local labor market  $m$  constant. Second, an establishment-occupation specific component which is enough

to characterize the local equilibrium as shown in Proposition 2. We denote this second term as:

$$\tilde{w}_{io} = \left( \beta_b \lambda(\mu_{io}, \varphi_b) \frac{A_{io}}{(T_{io} \Gamma_b^\eta)^\delta} \right)^{1/1+\varepsilon_b \delta}, \quad (\text{A3})$$

where  $\tilde{w}_{io}$  is a function of the employment shares of all the establishment-occupations in  $m$ . The wage is:  $w_{io} = \tilde{w}_{io} \Phi_m^{(1-\eta/\varepsilon_b)\delta/1+\varepsilon_b \delta} P_b^{1/(1-\alpha_b)(1+\varepsilon_b \delta)} \left(\frac{\Phi}{L}\right)^{\delta/1+\varepsilon_b \delta}$ . Using the definition of  $\Phi_m \equiv \sum_{io \in \mathcal{I}_m} T_{io} w_{io}^{\varepsilon_b}$ :

$$\Phi_m = \tilde{\Phi}_m^{1+\varepsilon_b \delta/1+\eta \delta} P_b^{\varepsilon_b/(1-\alpha_b)(1+\eta \delta)} \left(\frac{\Phi}{L}\right)^{\varepsilon_b \delta/1+\eta \delta}, \quad \tilde{\Phi}_m \equiv \sum_{io \in \mathcal{I}_m} T_{io} \tilde{w}_{io}^{\varepsilon_b}, \quad (\text{A4})$$

where  $\tilde{\Phi}_m$  is a function of the local labor market equilibrium  $\{s_{io|m}\}_{io \in \mathcal{I}_m}$  that can be solved separated from aggregates as shown in Proposition 2. Plugging  $\Phi_m$  into the wage,

$$w_{io} = \tilde{w}_{io} \tilde{\Phi}_m^{\frac{(\varepsilon_b - \eta)\delta}{\varepsilon_b(1+\eta \delta)}} P_b^{\frac{1}{(1-\alpha_b)(1+\eta \delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\eta \delta}}. \quad (\text{A5})$$

The establishment-occupation labor supply is  $L_{io} = s_{io|m} s_{m|b} L_b$ . Given the normalized wages per sub-market  $\tilde{w}_{io}$ , we can compute the employment share within the local labor market and the share of  $m$  out of the sector using the definition of  $\Phi_b \equiv \sum_{m \in \mathcal{M}_b} \Phi_m^{\eta/\varepsilon_b}$  and (A4):

$$s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} = \frac{T_{io} \tilde{w}_{io}^{\varepsilon_b}}{\tilde{\Phi}_m}, \quad \tilde{\Phi}_m \equiv \sum_{i \in \mathcal{I}_m} T_{io} \tilde{w}_{io}^{\varepsilon_b},$$

$$s_{m|b} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_b} = \frac{\tilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b \delta)}{\varepsilon_b(1+\eta \delta)}}}{\tilde{\Phi}_b}, \quad \tilde{\Phi}_b \equiv \sum_{m \in \mathcal{M}_b} \tilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b \delta)}{\varepsilon_b(1+\eta \delta)}},$$

where  $\mathcal{M}_b$  is the set of all local labor markets that belong to sector  $b$ . Knowing the relative wages within a sector, we can compute the measure of workers that go to each establishment, conditioning on sector employment. Using (A4), sector labor supply is a function of aggregators of 'tilde' variables  $\tilde{\Phi}_b(\mathbf{s}_b)$ , where  $\mathbf{s}_b \equiv \{s_{io|m}\}_{io \in \mathcal{I}_b}$ , and prices:

$$L_b = \frac{\Phi_b \Gamma_b^\eta}{\sum_{b' \in \mathcal{B}} \Phi_{b'} \Gamma_{b'}^\eta} L = \frac{P_b^{\eta/(1-\alpha_b)(1+\eta \delta)} \tilde{\Phi}_b(\mathbf{s}_b) \Gamma_b^\eta}{\tilde{\Phi}} L, \quad \tilde{\Phi} \equiv \sum_{b' \in \mathcal{B}} P_{b'}^{\eta/(1-\alpha_{b'})(1+\eta \delta)} \tilde{\Phi}_{b'}(\mathbf{s}_{b'}) \Gamma_{b'}^\eta. \quad (\text{A6})$$

This is where the simplifying assumption on the labor demand elasticity  $\delta \equiv 1 - \beta_b/1-\alpha_b$  being

constant across industries buys us tractability. We can factor out the economy wide constant from (A4) and leave everything in terms of normalized wages and transformed prices.

Finding equilibrium allocations requires solving the transformed prices  $\mathbf{P} = \{P_b\}_{b=1}^{\mathcal{B}}$ . Using the intermediate input demand (3) and the above expression for sector labor supply  $L_b$  we get:

$$P_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}} A_b \Omega_b \left( \tilde{\Phi}_b \Gamma_b^\eta \right)^{1-\delta} = \theta_b \prod_{b' \in \mathcal{B}} \left( A_{b'} \Omega_{b'} \left( \tilde{\Phi}_{b'} \Gamma_{b'}^\eta \right)^{1-\delta} \right)^{\theta_{b'}} \prod_{b' \in \mathcal{B}} \left( P_{b'}^{\frac{\alpha_{b'}(1+\eta\delta) + \eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}} \right)^{\theta_{b'}}. \quad (\text{A7})$$

Define  $f_b \equiv 1/1-\alpha_b \log(P_b)$  and  $\mathbf{f}$  as a  $B \times 1$  vector whose element  $b'$  is  $f_{b'}$ . Then, taking logs and rearranging the previous expressions for all  $b \in \mathcal{B}$  we obtain:

$$\mathbf{f} = \mathbf{C} + \mathbf{D}\mathbf{f}, \quad (\text{A8})$$

where  $\mathbf{C}$  is a  $B \times 1$  vector whose  $b$  element is

$$(\mathbf{C})_b = \frac{1+\eta\delta}{1+\eta} \left[ \log \left( \frac{\theta_b}{A_b \Omega_b} \right) - (1-\delta) \log \left( \tilde{\Phi}_b \Gamma_b^\eta \right) + \sum_{b' \in \mathcal{B}} \theta_{b'} \left( \log(A_{b'} \Omega_{b'}) + (1-\delta) \log(\tilde{\Phi}_{b'} \Gamma_{b'}^\eta) \right) \right],$$

and  $\mathbf{D}$  is a  $B \times B$  matrix whose  $b$  row  $b'$  column element is:

$$(\mathbf{D})_{bb'} = \frac{(\alpha_{b'}(1+\eta\delta) + \eta(1-\delta)) \theta_{b'}}{1+\eta}.$$

A solution to the system (A8) exists and is unique if the matrix  $\mathbf{I} - \mathbf{D}$  is invertible. This matrix has an eigenvalue of zero, and therefore is not invertible, if and only if  $\mathbf{D}$  has an eigenvalue equal to one.<sup>2</sup> The matrix  $\mathbf{D}$  has an eigenvalue equal to one if and only if the sum of the elements of the rows in matrix  $\mathbf{D}$  are equal to 1. To see this, let  $\mathbf{v}$  be the eigenvector associated with the unit eigenvalue of  $\mathbf{D}$ , i.e.  $\mathbf{D}\mathbf{v} = \mathbf{v}$ . If  $\mathbf{v} = \mathbf{1}$ , then, by the Perron-Frobenius theorem, it is the only eigenvector (up-to-scale) associated with the unit eigenvalue. Furthermore, if  $\mathbf{v} = \mathbf{1}$ , then  $\sum_{b'} (D)_{bb'} = 1$  for all  $b \in \mathcal{B}$ . Conversely, if  $\sum_{b'} (D)_{bb'} = 1$  for all  $b \in \mathcal{B}$ , then  $\mathbf{v} = \mathbf{1}$  is a solution for the eigensystem  $\mathbf{D}\mathbf{v} = \mathbf{v}$ . But, by the Perron-Frobenius theorem,  $\mathbf{v} = \mathbf{1}$  is the unique (up-to-scale) eigenvector associated with the unit eigenvalue. Therefore, the matrix  $\mathbf{I} - \mathbf{D}$  is not invertible if and only if the sum of the elements of the rows in matrix  $\mathbf{D}$  are equal to 1.

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<sup>2</sup>Proof: If 1 is an eigenvalue of  $\mathbf{D}$ , then  $\mathbf{D}\mathbf{v} = \mathbf{v}$  for a nonzero vector  $\mathbf{v}$ . Then  $(\mathbf{I} - \mathbf{D})\mathbf{v} = 0$ , so 0 is an eigenvalue of  $\mathbf{I} - \mathbf{D}$  with the associated eigenvector  $\mathbf{v}$ . Conversely, if 0 is an eigenvalue of  $\mathbf{I} - \mathbf{D}$ , then  $\mathbf{D}\mathbf{v} = \mathbf{v}$  and 1 is an eigenvalue of  $\mathbf{D}$ .

This sum is equal to 1 if and only if  $\sum_b \alpha_b \theta_b = 1$  as:

$$\begin{aligned} \sum_{b'} (\mathbf{D})_{bb'} = 1 &\Leftrightarrow \sum_{b'} (\alpha_{b'}(1 + \eta\delta) + \eta(1 - \delta)) \theta_{b'} = 1 + \eta \\ &\Leftrightarrow \sum_{b'} \alpha_{b'} \theta_{b'} = \frac{1 + \eta - \eta(1 - \delta)}{1 + \eta\delta} \Leftrightarrow \sum_b \alpha_b \theta_b = 1. \end{aligned}$$

Therefore whenever  $\sum_b \alpha_b \theta_b \neq 1$ ,  $\mathbf{f}$  has a unique solution. Also, if  $\alpha_b \neq 1$  for all  $b \in \mathcal{B}$ , then the vector of prices  $[P_b]_{b \in \mathcal{B}}$  has a unique solution as well. Solving for  $P_b$  in (A7):

$$P_b = X_b X^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, X_b = \left( \frac{\theta_b}{A_b \Omega_b (\tilde{\Phi}_b \Gamma_b^\eta)^{(1-\delta)}} \right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, X = \left( \prod_{b' \in \mathcal{B}} \left( \frac{\theta_{b'}}{X_{b'}} \right)^{\theta_{b'}} \right)^{\frac{1+\eta}{(1+\eta\delta) \sum_{b' \in \mathcal{B}} \theta_{b'} (1-\alpha_{b'})}}, \quad (\text{A9})$$

for all  $b \in \mathcal{B}$  where we used the aggregate price index  $1 = \prod_{b \in \mathcal{B}} \left( \frac{P_b}{\theta_b} \right)^{\theta_b}$  to find the economy wide constant  $X$ . The above is the closed-form solution of prices in Proposition 5.  $\square$

The sector price  $P_b$  depends positively on the final good elasticity  $\theta_b$ , reflecting that a higher demand for goods of sector  $b$  will increase its price. It also negatively depends on the product of productivity and misallocation  $A_b \Omega_b$  and the labor supply shifter for sector  $b$ ,  $\Gamma_b$ . An increase in any of both terms translates into more supply of sector  $b$  goods, either by being more productive or by increasing the labor employed in sector  $b$ . This in turn would reduce its price.

## B Derivations

In this section we provide the key derivations of the model that are not presented in the main text. The Supplemental Material covers additional derivations.

### B.1 Bargaining

We provide derivations under the baseline bargaining protocol where employers and unions have zero outside options and an alternative leading to the same equilibrium wages.

Each establishment has different occupation profit functions  $(1 - \alpha_b) P_b F(L_{io}) - w_{io}^u L_{io}$ , where the optimal capital decision has been taken. We assume that workers and establishments are

symmetric both having null threat points and internalizing the generation of rents as they move along the labor supply curve. Establishments and unions choose wages to maximize:

$$\max_{w_{io}^u} [w_{io}^u L_{io}(w_{io}^u)]^{\varphi_b} [(1 - \alpha_b)P_b F(L_{io}(w_{io}^u)) - w_{io}^u L_{io}(w_{io}^u)]^{1 - \varphi_b},$$

where  $\varphi_b$  is the union bargaining power,  $w_{io}^u$  is the bargained wage at establishment-occupation,  $L_{io}$  is the number of workers employed at  $io$  in equilibrium,  $(1 - \alpha_b)F(L_{io})$  is the output. The F.O.C. is:

$$\varphi_b \frac{(1 - \alpha_b)P_b F(L_{io}) - w_{io}^u L_{io}}{w_{io}^u L_{io}} \left[ L_{io} + w_{io}^u \frac{\partial L_{io}}{\partial w_{io}^u} \right] + (1 - \varphi_b) \left[ (1 - \alpha_b)P_b \frac{\partial F(L_{io})}{\partial L_{io}} \frac{\partial L_{io}}{\partial w_{io}^u} - L_{io} - w_{io}^u \frac{\partial L_{io}}{\partial w_{io}^u} \right] = 0.$$

Using the definition of the perceived labor supply elasticity  $e_{io} = \frac{\partial L_{io}}{\partial w_{io}^u} \frac{w_{io}^u}{L_{io}}$  and rearranging:

$$w_{io}^u = \varphi_b (1 - \alpha_b)P_b \frac{F(L_{io})}{L_{io}} + (1 - \varphi_b)(1 - \alpha_b)P_b \frac{\partial F(L_{io})}{\partial L_{io}} \frac{e_{io}}{e_{io} + 1},$$

where  $\mu(s_{io}) \equiv \frac{e_{io}}{e_{io} + 1}$  is the markdown (Proposition 1). Substituting the optimal decision for capital, the output elasticity of labor is  $1 - \delta$  so  $\frac{1}{1 - \delta} \frac{\partial F(L_{io})}{\partial L_{io}} = \frac{F(L_{io})}{L_{io}}$ . The bargained wage is:

$$w_{io}^u = \underbrace{(1 - \alpha_b)P_b \frac{\partial F(L_{io})}{\partial L_{io}}}_{MRPL_{io}} \left[ (1 - \varphi_b) \frac{e_{io}}{e_{io} + 1} + \varphi_b \frac{1}{1 - \delta} \right],$$

where we recovered the expression from the main text.

**Alternative bargaining protocol.** The alternative bargaining assumption leading to the same equilibrium wages is that employers and unions bargain over wages without internalizing movements along the labor supply and workers' outside options are the oligopsonistic competition wages  $w_{io}^M$  under the allocation with the given equilibrium wages. This alternative protocol can be rationalized by a set up where firms have to pay workers before production starts at the oligopsonistic wage. Then, workers would force a negotiation where they would split the remaining rents after payments to capital. The bargaining problem would be:

$$\max_{w_{io}^u} [w_{io}^u L_{io} - w_{io}^M L_{io}]^{\varphi_b} [(1 - \alpha_b)P_b F(L_{io}) - w_{io}^u L_{io}]^{1 - \varphi_b}.$$



## B.2 Hat algebra

Here we show how to compute the counterfactuals in general equilibrium by using revenue productivities (TFPRs), which are a function of prices determined in general equilibrium, and not just the underlying physical productivities. A priori, the issue is that counterfactually changing the labor wedge changes equilibrium prices and therefore the ‘fundamental’ TFPRs.

The literature on misallocation has used the TFPRs, together with a modeling assumption on the sector price, to compute the normalized within sector productivity distribution. This has prevented performing general equilibrium counterfactuals that also take into account productivity differences across industries.<sup>3</sup> We show that we can: (i) do counterfactuals in general equilibrium by writing the model relative to a baseline scenario; and (ii) compute the movement of production factors across sectors.

Our approach is to write counterfactual sector prices relative to the baseline and to fix the transformed revenue productivities  $Z_{io}$ .<sup>4</sup> From the definition of  $Z_{io} = P_b^{\frac{1}{1-\alpha_b}} A_{io}$  and equation (9), wages are:  $w_{io} = \beta_b \lambda(\mu_{io}, \varphi_b) Z_{io} L_{io}^{-\delta}$ . Denoting with a prime the variables in the counterfactual (e.g.  $P'_b$ ) and with a hat the relative variables (e.g.  $\hat{P}_b = \frac{P'_b}{P_b}$ ). The counterfactual revenue productivity is a function of the relative price  $\hat{P}_b$  and the observed revenue productivity  $Z_{io}$ . Let  $\lambda'_{io}$  be the counterfactual wedge, using the definition of the transformed TFPRs the counterfactual wages are:  $w'_{io} = \beta_b \lambda'_{io} Z'_{io} L'^{-\delta}_{io} = \beta_b \lambda'_{io} Z_{io} \hat{P}_b^{\frac{1}{1-\alpha_b}} L'^{-\delta}_{io}$ . In the counterfactuals  $Z_{io}$  is taken as a fixed fundamental and we have to solve for sector prices relative to the baseline  $\hat{P}_b$ . The system (A1) in the counterfactual is:

$$w'_{io} = \omega_{io} \left( \hat{P}_b^{\frac{1}{1-\alpha_b}} \right)^{\frac{1}{1+\varepsilon_b \delta}} \Phi'^{\frac{\delta(\varepsilon_b - \eta)}{\varepsilon_b(1+\varepsilon_b \delta)}} \left( \frac{\Phi'}{L} \right)^{\frac{\delta}{1+\varepsilon_b \delta}}, \quad (\text{B1})$$

where the establishment-occupation component in the counterfactual is:  $\omega_{io} \equiv (\beta_b \lambda'_{io} Z_{io} / (T_{io} \Gamma_b^\eta)^\delta)^{1/(1+\varepsilon_b \delta)}$ .

<sup>3</sup>For example, Hsieh and Klenow (2009) conduct a counterfactual where they remove distortions at the firm level and compute the productivity gains at the *sector* level. The productivity gains are a result of factors of production reallocating to more productive firms *within* each sector. This allows them to compute a *partial* equilibrium effect on total factor productivity, i.e. keeping the production factors constant *across* industries. They cannot do the general equilibrium counterfactual as they can identify only relative productivity differences within each sector while normalizing average differences across industries. For more details, see equation (19) and the discussion below in their paper.

<sup>4</sup>Solving the counterfactuals in levels (Section 3) requires backing out the productivities which is possible by making some additional normalizations per sector. One could assume that the minimum physical productivity (TFP) is constant across industries and get rid of sector relative prices by normalizing the minimum TFP per sector.

Finally,  $\omega_{io}$  are enough to compute the employment shares, as shown in Propositions 2 and 4. Rewriting the employment counterfactual share as in Proposition 2 but with TFPRs:

$$s'_{io|m} = \frac{\left(T_{io}^{1/\varepsilon_b} \lambda'_{io} Z_{io}\right)^{\varepsilon_b/1+\varepsilon_b\delta}}{\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} \lambda'_{jo} Z_{jo}\right)^{\varepsilon_b/1+\varepsilon_b\delta}} = \frac{s_{io|m} (\lambda'_{io}/\lambda_{io})^{\varepsilon_b/1+\varepsilon_b\delta}}{\sum_{j \in \mathcal{I}_m} s_{jo|m} (\lambda'_{jo}/\lambda_{jo})^{\varepsilon_b/1+\varepsilon_b\delta}},$$

where we substituted the identified values for the revenue productivities  $Z_{io} = \frac{w_{io} L_{io}^\delta}{\beta_b \lambda_{io}}$  and amenities  $\frac{s_{io|m}}{w_{io}^{\varepsilon_b}} \left(\frac{L_m}{\Gamma_b}\right)^{\varepsilon_b/\eta}$ . See Section C.4 of this Online Appendix for the derivation. Therefore, it is equivalent to compute the counterfactual employment shares within a local labor market using the observed employment shares and wedges in the baseline, or the identified amenities and revenue productivities. We can then use the revenue productivities, which are a function of observed wages, employment levels and wedges to aggregate the counterfactual economy at the sector level. Following the same steps as in the baseline, the sector level system of equations in the counterfactual is analogous to (12) but with relative variables. Solving for relative sector prices we can compute the sector employment  $L'_b$ . Propositions 3 and 5 apply also in the 'hat' economy. The solution for the counterfactuals exists and is unique.

Summing the counterfactual wage  $w'_{io}$  from (B1) to  $\Phi'_m = \sum_{i \in \mathcal{I}_m} T_{io} w'_{io}{}^{\varepsilon_b}$  and factoring out the industry or economy wide constants we find the following relation:  $\Phi'_m = \widetilde{\Phi}'_m \frac{1+\varepsilon_b\delta}{1+\eta\delta} \widehat{P}_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi'}{L'}\right)^{\frac{\varepsilon_b\delta}{1+\eta\delta}}$ , where  $\widetilde{\Phi}'_m \equiv \sum_{i \in \mathcal{I}_m} T_{io} \omega_{io}^{\varepsilon_b}$ . Using the definition of  $\Phi'_b \equiv \sum_{m \in \mathcal{M}_b} \Phi'_m \eta/\varepsilon_b$  and  $\Phi' \equiv \sum_{b \in \mathcal{B}} \Phi'_b \Gamma_b^\eta$ , we have that:

$$\begin{aligned} \Phi'_b &= \widetilde{\Phi}'_b \widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi'}{L'}\right)^{\frac{\eta\delta}{1+\eta\delta}}, \quad \widetilde{\Phi}'_b \equiv \sum_{m \in \mathcal{M}_b} \widetilde{\Phi}'_m \frac{(1+\varepsilon_b\delta)\eta}{(1+\eta\delta)\varepsilon_b}, \\ \Phi' &= \widetilde{\Phi}'^{1+\eta\delta} L'^{-\eta\delta}, \quad \widetilde{\Phi}' \equiv \sum_{b \in \mathcal{B}} \widetilde{\Phi}'_b \widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \Gamma_b^\eta. \end{aligned}$$

Sector employment in the counterfactual is a function of relative prices  $\{\widehat{P}_b\}_{b \in \mathcal{B}}$  and counterfactual employment shares  $\{s'_b\}_{b \in \mathcal{B}}$ :  $L'_b = \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \widetilde{\Phi}'_b (s'_b) \Gamma_b^\eta}{\sum_{b' \in \mathcal{B}} \widehat{P}_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta\delta)}} \widetilde{\Phi}'_{b'} (s'_{b'}) \Gamma_{b'}^\eta} L'$ . Counterfactual output is:

$$y'_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} L'_{io}{}^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}} L'_{io}{}^{1-\delta} = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} Z_{io} L'_{io}{}^{1-\delta}.$$

The analogue expression for the baseline is:  $y_{io} = \frac{1}{P_b} Z_{io} L_{io}^{1-\delta}$ . To aggregate this expression note that the revenue productivities are multiplied by a sector-level constant and cancel out,

$$\Psi'_b \equiv \sum_{io \in \mathcal{I}_b} \frac{Z_{io}}{Z_b} s'_{io|m}{}^{1-\delta} s'_{m|b}{}^{1-\delta} = \sum_{io \in \mathcal{I}_b} \frac{A_{io}}{A_b} s'_{io|m}{}^{1-\delta} s'_{m|b}{}^{1-\delta} \equiv \Omega'_b,$$

where  $\Omega'_b$  is a measure of sector productivity in the counterfactual relative to the productivity under the efficient allocation  $A_b$ . Aggregating up to sector level, the counterfactual output is,

$$Y'_b = \frac{\hat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} Z_b \Omega'_b L_b^{1-\delta}, \quad \Omega'_b \equiv \sum_{io \in \mathcal{I}_b} \frac{A_{io}}{A_b} s'_{io|m}{}^{1-\delta} s'_{m|b}{}^{1-\delta}, \quad Z_b \equiv \sum_{io \in \mathcal{I}_b} Z_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta}.$$

The baseline sector output is:  $Y_b = 1/P_b Z_b \Omega_b L_b^{1-\delta}$  where  $\Omega_b$  is a function of baseline employment shares,  $\Omega_b \equiv \sum_{io \in \mathcal{I}_b} A_{io}/A_b s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}$ . Counterfactual output relative to the baseline is:

$$\hat{Y}_b = \hat{P}_b^{\frac{\alpha_b}{1-\alpha_b}} \hat{\Omega}_b \hat{L}_b^{1-\delta}, \quad (\text{B2})$$

where  $\hat{\Omega}_b = \Omega'_b/\Omega_b$ . Using  $L'_b$  and (3) we get a similar expression to (A7) canceling constants:

$$\hat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}} \hat{\Omega}_b \left( \frac{\tilde{\Phi}'_b \Gamma_b^\eta}{L_b} \right)^{1-\delta} = \prod_{b' \in \mathcal{B}} \left( \hat{P}_{b'}^{\frac{\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}} \right)^{\theta_{b'}} \prod_{b' \in \mathcal{B}} \hat{\Omega}_{b'}^{\theta_{b'}} \prod_{b' \in \mathcal{B}} \left( \frac{\tilde{\Phi}'_{b'} \Gamma_{b'}^\eta}{L_{b'}} \right)^{(1-\delta)\theta_{b'}}. \quad (\text{B3})$$

Rewriting, we get a similar expression to equation (A9) in Proposition 5 but with hat variables:

$$\hat{P}_b = \hat{X}_b \hat{X}^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad \hat{X}_b = \left( \frac{L_b^{1-\delta}}{\hat{\Omega}_b (\tilde{\Phi}'_b \Gamma_b^\eta)^{1-\delta}} \right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad \hat{X} = \left( \prod_{b' \in \mathcal{B}} \hat{X}_{b'}^{-\theta_{b'}} \right)^{\frac{1+\eta}{\sum_{b' \in \mathcal{B}} \theta_{b'} (1-\alpha_{b'}) (1+\eta\delta)}}.$$

**Fixed labor.** Fixing employment at the sector level  $b$ , the counterfactual wage (B1) becomes:

$$w'_{io} = \left( \beta_b \lambda_{io} \frac{Z_{io}}{\Gamma_{io}^\delta} \right)^{\frac{1}{1+\varepsilon_b\delta}} \hat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \Phi'_m (1-\eta/\varepsilon_b)^{\frac{\delta}{1+\varepsilon_b\delta}} \left( \frac{\Phi'_b}{L'_b} \right)^{\frac{\delta}{1+\varepsilon_b\delta}}.$$

Fixing lower levels than  $b$  would only change the last element. Keeping total employment at the local labor market fixed, the last term would become:  $(\Phi'_m/L'_m)^{\delta/1+\varepsilon_b\delta}$ . The constant  $\Gamma_b$  does not

appear because workers can't move across sectors. Fixing lower levels than  $b$  clearly implies that  $L'_b$  is equal to  $L_b$ . Given that  $L'_b$  is known we have a condition similar to (B3):

$$\widehat{P}_b^{\frac{1}{1-\alpha_b}} \widehat{\Omega}_b = \prod_{b' \in \mathcal{B}} \left( \widehat{P}_{b'}^{\frac{\alpha_{b'}}{1-\alpha_{b'}}} \widehat{\Omega}_{b'} \right)^{\theta_{b'}}.$$

Propositions 3 and 5 also apply in hat counterfactuals with fixed labor at  $b$  or lower levels.

## C Identification and estimation

### C.1 Identification of common parameters $\eta$ and $\delta$

We identify the across markets labor supply elasticity  $\eta$  and the labor demand elasticity  $\delta$  by noticing that in local labor markets where there is only one establishment, the wedge  $\lambda(\mu, \phi_b)$  is constant within sectors. We denominate this type of establishments as *full monopsonists*. Taking the logarithm of the labor supply that full monopsonists face:  $\ln(L_{io,s=1}) = \eta \ln(w_{io}) + \ln(\tilde{T}_{io}) + \ln(\Gamma_b^\eta L / \Phi)$ , where  $\tilde{T}_{io} = T_{io}^{\eta/\varepsilon_b}$ . Full monopsonists apply a constant markdown equal to  $\mu(s=1) = \frac{\eta}{\eta+1}$  that in turn will imply a constant wedge  $\lambda(\mu, \phi_b)$  within industry  $b$ . Their inverse labor demand (9) in logs is:

$$\ln(w_{io,s=1}) = \ln(\beta_b) + \ln \left( (1 - \phi_b) \frac{\eta}{\eta+1} + \phi_b \frac{1}{1-\delta} \right) + \ln(A_{io}) - \delta \ln(L_{io}) + \frac{1}{1-\alpha_b} \ln(P_b).$$

To get rid of constants, we demean  $\ln(L_{io,s=1})$  and  $\ln(w_{io,s=1})$  by removing the sector  $b$  averages per year. Denoting with  $\overline{\ln(X)}$  the demeaned variables, we rewrite (13) and (14) as:

$$\overline{\ln(L_{io})} = \eta \overline{\ln(w_{io})} + \overline{\ln(\tilde{T}_{io})}, \quad \overline{\ln(w_{io})} = -\delta \overline{\ln(L_{io})} + \overline{\ln(A_{io})}. \quad (\text{C1})$$

The above system is a traditional demand and supply setting and suffers from simultaneity bias and is under-identified. The variance-covariance matrix of  $(\overline{\ln(L_{io})}, \overline{\ln(w_{io})})$  gives us three moments from the data, the system above has five unknowns, which are  $\eta$  and  $\delta$ , plus the three components of the variance-covariance matrix of the structural errors  $\overline{\ln(\tilde{T}_{io})}$  and  $\overline{\ln(A_{io})}$ . Therefore, in absence of valid instruments that would exogenously vary either the supply or demand equations in (C1) we can not identify the elasticities through exclusion restrictions.

We impose restrictions on the variance-covariance matrix of the structural errors while ex-

exploiting the differences in the variance-covariance matrix of the employment and wages across occupations. This way of achieving identification is known in the literature as *identification through heteroskedasticity* (Rigobon, 2003). We classify our four occupations into two broader categories  $S \in \{1,2\}$  which we denote as blue collar and white collar. Our identification assumption is that the covariance between the transformed productivity  $\overline{\ln(A_{io})}$  and amenities  $\overline{\ln(\tilde{T}_{io})}$ , that we denote  $\sigma_{TA}$ , is constant within each category  $S$ . Having the same elasticities across occupational groups within the categories, and the assumption of common covariance of the structural errors within broad categories, we can achieve identification. While the four occupations give us 12 moments, the unknowns are also 12:  $\delta$  and  $\eta$ , plus 2, the broad category covariances, plus 8, the variances of the transformed productivities and amenities for each of the four occupations. We can rewrite the system (C1) as:

$$\overline{\ln(\tilde{T}_{io})} = \overline{\ln(L_{io})} - \eta \overline{\ln(w_{io})}, \quad \overline{\ln(A_{io})} = \delta \overline{\ln(L_{io})} + \overline{\ln(w_{io})}. \quad (\text{C2})$$

Defining an auxiliary parameter  $\tilde{\delta} = -\delta$  and using our identifying assumption  $\sigma_{AT,oS} = \sigma_{AT,o'S} = \sigma_{AT,S}$  for occupations that belong to the same category  $S$ , the system is:

$$\begin{pmatrix} \sigma_{T,oS}^2 & \sigma_{TA,S} \\ \sigma_{TA,S} & \sigma_{A,oS}^2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -\eta \\ -\tilde{\delta} & 1 \end{pmatrix}}_D \underbrace{\begin{pmatrix} \sigma_{L,oS}^2 & \sigma_{LW,oS} \\ \sigma_{LW,oS} & \sigma_{W,oS}^2 \end{pmatrix}}_{\Omega_{oS}} \underbrace{\begin{pmatrix} 1 & -\tilde{\delta} \\ -\eta & 1 \end{pmatrix}}_{D^T}$$

This system only allows us to identify  $\eta$  and  $\delta$ . Denote by  $\Omega_S \equiv \Omega_{oS} - \Omega_{o'S}$  the difference between the variance covariance matrix of wages and employment within category  $S$ ,  $\Delta_S$  the difference in the covariance matrix of the structural shocks. Let  $\omega_{ij,S}$  be the element on  $i$ th row and  $j$ th column of  $\Omega_S$ . The system of differences is:  $\Delta_S = D\Omega_S D^T, \forall S \in \{1,2\}$ . With the identification assumption of equal covariance within category, we have that:

$$\Delta_{S,[1,2]} = 0 = -\eta\omega_{22,S} + (1 + \eta\tilde{\delta})\omega_{12,S} - \tilde{\delta}\omega_{11,S} \quad \Rightarrow \quad \eta = \frac{\omega_{12,S} - \tilde{\delta}\omega_{11,S}}{\omega_{22,S} - \tilde{\delta}\omega_{12,S}}, \quad \forall S \in \{1,2\}$$

Equalizing the above across both occupation categories we get a quadratic equation in  $\tilde{\delta}$ :

$$\tilde{\delta}^2[\omega_{11,1}\omega_{12,2} - \omega_{11,2}\omega_{12,1}] - \tilde{\delta}[\omega_{11,1}\omega_{22,2} - \omega_{11,2}\omega_{22,1}] + \omega_{12,1}\omega_{22,2} - \omega_{12,2}\omega_{22,1} = 0. \quad (\text{C3})$$

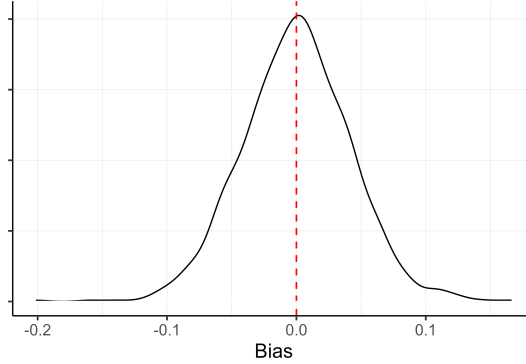
This is the same system as the simple case with zero covariance between the fundamental shocks in Rigobon (2003). Different to him,  $\Omega_S$  is not directly the estimated variance-covariance matrix of each of the 4 occupations but rather the matrix of covariance differences within category  $S$ . As mentioned by Rigobon (2003) there are two solutions. One can show that if  $\tilde{\delta}^*$  and  $\eta^*$  are a solution then the other solution is equal to  $\tilde{\delta} = 1/\eta^*$  and  $\eta = 1/\tilde{\delta}^*$ . We have that by assumption  $\eta$  is positive while  $\tilde{\delta}$  is negative. Therefore as long as the two possible solutions for  $\tilde{\delta}$  have different signs, we just need to pick the negative one.

## C.2 Validation of the identification of $\varepsilon_b$

In this section, we validate our identification strategy of the within-labor market labor supply elasticities via simulations. We perform 1,000 simulations of an economy populated with 200 local labor markets for 14 years. The number of competitors in the local labor market follows an exponential distribution with mean 4 and standard deviation of 1, and the logarithm of productivities and amenities are normally distributed with means of 1 for both and standard deviations of 0.8 and 0.1 respectively. Population is assumed to be symmetrically distributed across local labor markets. We simulate productivities, amenities and number of competitors in local labor markets of the *Food* sector. We solve for each local labor market independently of aggregates and characterize  $w_{io}^S = \left(T_{io}^{1/\varepsilon_b} \lambda_{io} A_{io}\right)^{1/1+\varepsilon_b\delta}$  for each establishment.

We estimate equation (15) in the simulated equilibrium by regressing the logarithm of employment on log wages. We control for the strategic interactions by introducing local labor market fixed effects and therefore only use within-local labor market variation to identify the local elasticity of substitution. Figure C1 presents the bias of the IV estimates when we instrument for contemporaneous log wages by a proxy of establishment revenue productivity:  $\hat{A}_{iot} = \frac{P_{bt} Y_{jt}}{L_{iot}^{1-\delta}}$ . The figure shows that even in the presence of amenities, which are labor supply shifters that correlate with wages, our identification strategy recovers the local elasticities of substitution as the density is centered around 0.

Figure C1: Bias of estimated  $\varepsilon_b$



Note: Density of the difference between the estimated local elasticity of substitution and the true parameter when simulating the model for sector 15 *Food*.

### C.3 Identification of $\varphi_b$

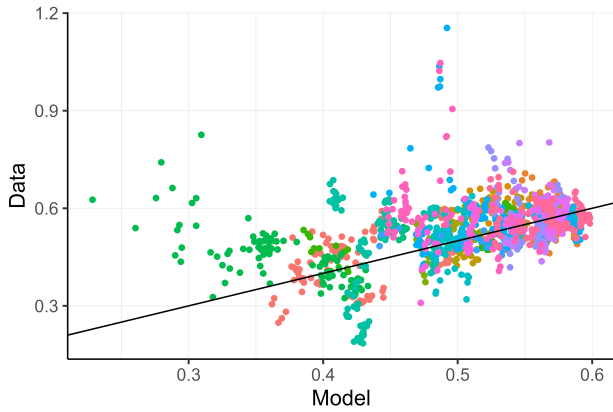
We identify the sector specific workers bargaining power by constructing the model counterparts of the sector labor share per year  $t$ :  $LS_{bt}^M(\varphi_b) = \frac{\beta_b \sum_{io \in \mathcal{I}_b} w_{iot} L_{iot}}{\sum_{io \in \mathcal{I}_b} w_{iot} L_{iot} / \lambda(\mu_{io}, \varphi_b)}$ . We target the average across time industry labor share such that:  $\mathbb{E}_t [LS_{bt}^M(\varphi_b) - LS_{bt}^D] = 0$ , where  $LS_{bt}^D$  is the labor share of sector  $b$  at time  $t$  that we observe in the data. The wedge  $\lambda(\mu_{io}, \varphi_b)$  is increasing in  $\varphi_b$ , and  $LS_{bt}^M(\varphi_b)$  is increasing in  $\varphi_b$  as well. Therefore, if a solution exists with  $\varphi_b \in [0, 1]$  it is unique.

### C.4 Amenities

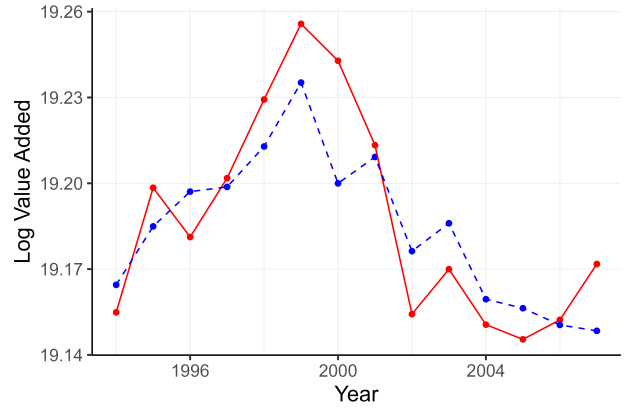
We still need to recover establishment-occupation amenities while ensuring that in equilibrium the wages and labor allocations are the same as in the data. Using the labor supply (7), we can back out amenities, up to a constant:  $T_{io} = \frac{s_{io|m}}{(w_{io})^{\varepsilon_b}} \Phi_m$ . We proceed by normalizing one local labor market as the allocation of resources is independent from this normalization. We denote the market that we normalize as 1. The relative employment share of market  $m$  with respect to the normalized one is:  $\frac{L_m}{L_1} = \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b}{\Phi_1^{\eta/\varepsilon_b} \Gamma_1}$ . The local labor market aggregate is then:

$$\Phi_m = \left( \frac{L_m \Gamma_1}{L_1 \Gamma_b} \Phi_1^{\eta/\varepsilon_b} \right)^{\varepsilon_b/\eta}. \text{ Substituting into the amenity: } T_{io} \propto \frac{s_{io|m}}{(w_{io})^{\varepsilon_b}} \left( \frac{L_m}{\Gamma_b} \right)^{\varepsilon_b/\eta}.$$

Figure C2: Model Fit Non Targeted Moments



(a) Sub-industry labor share



(b) Aggregate VA. Model (dashed blue), data (red).

## C.5 Non targeted moments

Panel (a) of Figure C2 presents 3-digit industry labor shares per year. The horizontal axis shows model generated moments, while the vertical axis has the corresponding observed moment in the data. If the fit was perfect, each dot would be on the 45 degree line. Each color represents a 2-digit industry. We see that most of the dots are aligned around the 45 degree line.

Panel (b) shows the model matches the evolution of aggregate value added. Since there is a strong link between production and wage bill and the model matches wages and labor allocations exactly, it also has a good fit of the value added.

## D Counterfactuals

We present additional results on the implications of labor market power on urban-rural differences and model extensions allowing for endogenous labor force participation and agglomeration forces.

### D.1 The effect of labor market power on urban-rural differences

Figure 4 suggests an important labor reallocation from rural areas to cities in the counterfactual without unions. This section explores the impact of employer and union labor market power



Table D1: Wage Gap

	Rural Wage	Urban Wage	Gap (%)
Baseline	33.319	45.210	36
Counterfactual. Oligopsony	24.592	36.861	50
Counterfactual. No wedges	49.486	60.675	23

*Note:* Wages in constant 2015 euros. *Urban:* 10 biggest commuting zones: Paris, Marseille, Lyon, Toulouse, Nantes, and the Paris surrounding areas of Boulogne-Billancourt, Creteil, Montreuil, Saint-Denis and Argenteuil. The rest are considered as *Rural*. Wages are employment weighted averages per location in 2007.

on the urban-rural wage gap.<sup>5</sup> Table D1 presents wage levels and the urban-rural wage gap. Both urban and rural areas experience important wage changes in the counterfactuals. Under oligopsonistic competition, the urban-rural wage gap amplifies from 36% to 50% and is cut up to 23% without labor wedges. This reveals that labor market distortions account for more than a third of the urban-rural wage gap.

## D.2 Extensions

The main counterfactual assumed that the total labor supply was fixed and there were no agglomeration externalities. Here we present results from counterfactuals that relax these assumptions allowing for an endogenous labor participation decision, and for agglomeration forces. All the details are left for the [Supplemental Material](#).

### D.2.1 Endogenous labor force participation

In the extension with endogenous labor force participation decisions we assume workers can decide between working and staying at home. In the latter case, they earn wages related to home production which is now an endogenous choice.

Table D2 shows the results of the counterfactuals with endogenous labor force participation. Introducing this margin induces higher output losses than in the baseline (*Fixed L*). The counterfactual output change without unions is  $-1.42\%$  as the total labor supply decreases by  $0.98\%$ . In contrast to the output decomposition in Table 4, 40% of the losses come from employment. This extensive margin of adjustment in the total labor supply amplifies the original differences

<sup>5</sup>In the [Supplemental Material](#) we further explore the effect on employment changes over time.

Table D2: Counterfactual: Endogenous Participation

	$\Delta Y$ (%)	$\Delta$ Prod (%)	$\Delta$ L (%)	Contribution $\Delta Y$ (%)		
				GE	Prod	Labor
<i>Fixed L</i>	-0.48	-0.95		-20	200	-80
<i>Endogenous participation (EP)</i>						
Oligopsony $\lambda(\mu, 0) = \mu_{io}$	-1.42	-0.95	-0.98	-7	67	40
No wedges $\lambda(1, 0) = 1$	2.61	1.33	1.01	6	52	42
Bargain $\lambda(1, \varphi_b) = 1 + \varphi_b \frac{\delta}{1-\delta}$	2.63	1.33	1.06	6	51	43
<i>EP Mobility within sector</i>						
Oligopsony $\lambda(\mu, 0) = \mu_{io}$	-1.86	-0.95	-1.02	-1	51	50

Notes: Results in percentages. The first three columns are changes relative to the baseline.  $\Delta Y$ : aggregate output,  $\Delta$  Prod: aggregate productivity from decomposition (18),  $\Delta L$ : aggregate employment. Last three columns present the contribution from decomposition (18) to output gains. *Fixed L*: main counterfactual with oligopsonistic competition, under free mobility of labor and fixed total labor supply. All the other counterfactuals in this table allow for endogenous labor force participation. *Oligopsony*: the first instance allows for free mobility of workers while the second one keeps sector workers (employed and unemployed) constant, *No wedges*: wedges equal to 1 (perfect competition), *Bargain*: standard bargaining framework.

in output gains across counterfactuals. In particular, output gains without labor wedges are as high as 2.61% because total labor force participation is increased by 1.01%. Despite featuring high wage changes, the differences in total employment are minor in the counterfactuals because we assume that workers have idiosyncratic shocks to stay out of the labor force.

Table D2 shows that sector reallocation contributes to the negative output effects of oligopsonistic competition as it constitutes 40% of the output loss. When fixing total sectoral labor force—employed and unemployed—output changes by  $-1.86\%$ , which is roughly 30 percent higher than the free mobility counterfactual.

## D.2.2 Agglomeration

We extend the model to include agglomeration forces at the local labor market level. We assume that the agglomeration effect is a local labor market externality with elasticity  $\gamma(1 - \alpha_b)$ .

Table D3 summarizes the counterfactuals for different values of  $\gamma$  under oligopsonistic competition, free mobility and fixed total employment. As  $\gamma$  becomes higher, the more important are the agglomeration forces and the more contained are the output losses. The reason behind this result is that increasing  $\gamma$  the local labor market employment  $L_m$  becomes more important in equation (I4). Consequently, differences in baseline employment levels across local labor

Table D3: Counterfactuals: Agglomeration. Oligopsony

	$\Delta Y$ (%)	$\Delta Prod$ (%)	Contribution $\Delta Y$ (%)		
			GE	Productivity	Labor
<i>No Agglomeration</i>	-0.48	-0.95	-20	200	-80
<i>Agglomeration</i>					
$\gamma = 0.05$	-0.47	-0.99	-16	209	-92
$\gamma = 0.1$	-0.46	-1.02	-13	219	-106
$\gamma = 0.2$	-0.45	-1.08	-4	240	-136
$\gamma = 0.25$	-0.44	-1.11	1	252	-154
$\gamma = 0.3$	-0.43	-1.13	8	265	-172

*Notes:* Results in percentages.  $\Delta Y$ : change of aggregate output relative to the baseline,  $\Delta Prod$ : change in aggregate productivity from decomposition (18). Last three columns present the contribution from decomposition (18) to output gains. *No Agglomeration*: main counterfactual under oligopsonistic competition, free mobility of labor, fixed total labor supply and no agglomeration forces. All the other counterfactuals in this table allow for agglomeration within the local labor market while workers are freely mobile and total employment is fixed. We present different counterfactuals depending on the agglomeration parameter  $\gamma$ .

markets amplify their productivity differences. [Supplemental Material IV](#) presents the agglomeration counterfactuals without labor wedges where baseline output gains are amplified.

## E Empirical evidence

Here we provide the link between the reduced form and our structural framework. We also present additional results, robustness checks and results on rent sharing elasticities.

### E.1 Labor market power and wages

#### E.1.1 Instrument: Mass layoff shock

The mass layoff shock instrument intends to capture the effect of a negative idiosyncratic productivity shock on close competitors. To provide some intuition, it will be helpful to focus on a local labor market with only 2 competitors. Using Proposition 2 and getting rid of subscript  $o$  and assuming constant amenities for simplicity, the employment share of establishment 1 is:

$$s_{1|m} = \frac{\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} + \left(\lambda(1-s_{1|m})A_2\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}, \quad (\text{E1})$$

where the denominator is the aggregator  $\Phi_m$ . This equation completely characterizes the equilibrium in  $m$ :  $\{s_{1|m}, 1 - s_{1|m}\}$ .

The above implicitly defines  $s_{1|m}$  as a function of  $A_2$  and  $\lambda(g(s_{1|m}))$ , where  $g(s_{1|m}) = s_{1|m}$  or  $g(s_{1|m}) = 1 - s_{1|m}$ . We can represent the system as:  $F(s_{1|m}, A_2, \lambda(g(s_{1|m})))$ . Using the implicit function theorem we have that:  $ds_{1|m}/dA_2 = -\frac{\partial F(\cdot)}{\partial A_2} / \frac{\partial F(\cdot)}{\partial s_{1|m}}$ . Developing the partial derivatives:

$$\begin{aligned} \frac{\partial F(\cdot)}{\partial A_2} &= -\Phi_m^2 \frac{\varepsilon_b}{1 + \varepsilon_b \delta} \lambda(1 - s_{1|m})^{\frac{\varepsilon_b}{1 + \varepsilon_b \delta}} A_2^{\frac{\varepsilon_b}{1 + \varepsilon_b \delta} - 1} < 0, \\ \frac{\partial F(\cdot)}{\partial s_{1|m}} &= \Phi_m^{-2} \frac{\varepsilon_b}{1 + \varepsilon_b \delta} \frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}} \left( \lambda(s_{1|m}) A_1 \right)^{\frac{\varepsilon_b}{1 + \varepsilon_b \delta}} \lambda(s_{1|m})^{-1} \Phi_m - \left( \lambda(s_{1|m}) A_1 \right)^{\frac{\varepsilon_b}{1 + \varepsilon_b \delta}} \frac{\varepsilon_b}{1 + \varepsilon_b \delta} \Phi_m^{-2} \\ &\quad \left[ \left( \lambda(s_{1|m}) A_1 \right)^{\frac{\varepsilon_b}{1 + \varepsilon_b \delta}} \lambda(s_{1|m})^{-1} \frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}} - \left( \lambda(1 - s_{1|m}) A_2 \right)^{\frac{\varepsilon_b}{1 + \varepsilon_b \delta}} \lambda(1 - s_{1|m})^{-1} \frac{\partial \lambda(1 - s_{1|m})}{\partial (1 - s_{1|m})} \right] - 1 \\ &= \frac{\varepsilon_b}{1 + \varepsilon_b \delta} s_{1|m} (1 - s_{1|m}) \left\{ \frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}} \lambda(s_{1|m})^{-1} + \lambda(1 - s_{1|m})^{-1} \frac{\partial \lambda(1 - s_{1|m})}{\partial (1 - s_{1|m})} \right\} - 1 < 0, \end{aligned}$$

where we used (E1) and the fact that  $\frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}} < 0$  and  $\frac{\partial \lambda(1 - s_{1|m})}{\partial (1 - s_{1|m})} < 0$ . We therefore have that:

$\frac{ds_{1|m}}{dA_2} < 0$ . Abstracting from market level constants,  $\log(w_1) = \left( \lambda(s_{1|m}) A_1 \right)^{\frac{1}{1 + \varepsilon_b \delta}}$ . We have:

$$\begin{aligned} \frac{d \log(w_1)}{dA_2} &= \frac{\partial \log(w_1)}{\partial A_2} + \frac{\partial \log(w_1)}{\partial s_{1|m}} \frac{ds_{1|m}}{dA_2} \\ &= 0 + \underbrace{\frac{\partial \log(w_1)}{\partial \log(\lambda(s_{1|m}))}}_{>0} \underbrace{\frac{\partial \log(\lambda(s_{1|m}))}{\partial s_{1|m}}}_{<0} \underbrace{\frac{ds_{1|m}}{dA_2}}_{<0} > 0. \end{aligned}$$

When a shock occurs to a competitor's productivity, the covariance between employment shares and log wages becomes negative. Using an IV regression, we identify the reduced-form effect which is different from the structural estimate holding everything else constant. As explained in Section 4.2 of the main text, strategic interactions can trigger responses from other market participants, which changes the underlying environment. However, as explained by Berger et al. (2022), the reduced-form estimate is still informative of the structural response.

**Definition of a mass layoff.** The definition of a mass layoff is firm-occupation specific. Denote by  $ML$  the set of firm-occupations with a *national* mass layoff. That is, firm-occupations with all the establishments suffering a mass layoff. Defining a cut-off value  $\kappa$ , we identify a firm-

occupation  $j \in ML$  if:  $L_{io,t}/L_{io,t-1} < \kappa \forall i$  belonging to firm  $j$ . A local labor market is identified as shocked  $D_{m,t} = 1$  if at least one establishment at the local market belongs to a firm in  $ML$ .

We instrument the employment share of the establishments of firm-occupations in  $j \notin ML$  by the exogenous event of a firm in the local labor market having a negative shock. We restrict the analysis to non-shocked multi-location firm-occupations with at least one establishment in a market where a competitor has suffered a mass layoff and another establishment whose competitors do not belong to firms in  $ML$ . The first stage is:

$$s_{io|m,t} = \psi_{J(i),o,t} + \delta_{N(i),t} + \gamma D_{m,t} + \epsilon_{io,t}$$

where  $\psi_{J(i),o,t}$  is a firm-occupation-year fixed effect and  $\delta_{N(i),t}$  is a commuting zone-year fixed effect. Using the fitted values, the second stage is:

$$\log(w_{io,t}) = \psi_{J(i),o,t} + \delta_{N(i),t} + \alpha \widehat{s_{io|m,t}} + u_{io,t} \quad (E2)$$

### E.1.2 Robustness checks

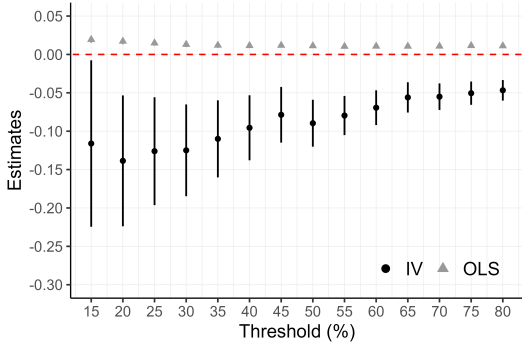
This section presents robustness checks of the reduced form evidence. First, we consider a different instrument for the employment shares and we change the main specification by taking commuting zone fixed effects. The results in the main text are with commuting zone-year fixed effects. Second, we present a robustness check to a different definition of local labor markets.

**Instrument.** Panel (a) of Figure E1 shows results when the new instrument is not binary and takes into account the original employment share of the mass layoff establishments. Panel (b) of the same figure shows the results using the main text specification but with commuting zone fixed effects. Results are qualitatively unchanged from the baseline in both cases.

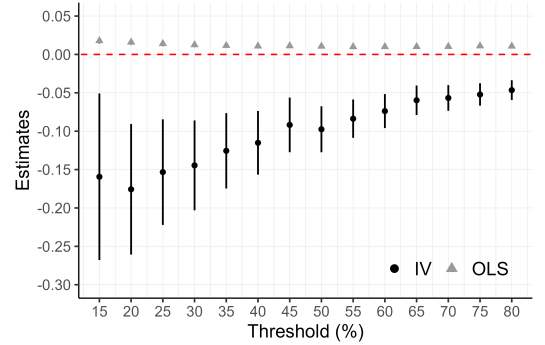
We build an additional instrument by lagged concentration measures. Table V1 in the [Supplemental Material](#) shows the results that qualitatively are similar to the baseline results.

**Controlling for labor demand.** When there are decreasing returns to scale, establishments labor demand has a negative slope. Thus, an increase in the employment level would imply a movement along the labor demand curve and lead to wage reductions. To take into account the potential effects of changes along the labor demand curve after the mass-layoff shock, we

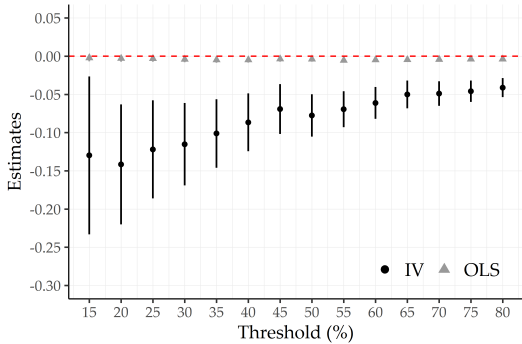
Figure E1: Robustness



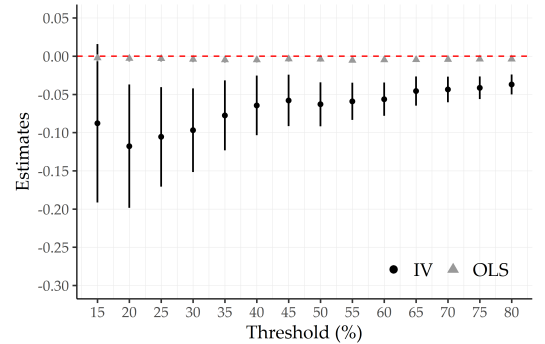
(a) Instrument: Intensive Share



(b) CZ fixed effects



(c) Control  $\log(L_{i,o,t})$



(d) Control  $\log(L_{i,o,t})$  and instrument

Notes: Point estimates and 95% confidence bands of the OLS and IV exercises on the y-axis. The x-axis presents different thresholds  $\kappa$  that define a mass layoff shock. In all cases we focus on non-affected competitors (not suffering a mass layoff shock). The instrument in Panel (a) is the presence of a mass layoff shock firm in the local labor market interacted with the employment share of the affected firm. Panel (b) presents the results with commuting zone fixed effects. For Panels (c) and (d) the specification is equation (E3). The figure in Panel (c) controls directly for  $\log(L_{i,o,t})$ , while Panel (d) instruments the logarithm of employment with its lagged value.

control for the employment level as in the following model:

$$\log(w_{i,o,t}) = \beta s_{i,o|m,t} + \gamma \log(L_{i,o,t}) + \psi_{J(i),o,t} + \delta_{N(i),t} + \epsilon_{i,o,t}, \quad (\text{E3})$$

where  $\log(w_{i,o,t})$  is the log average wage at plant  $i$  of firm  $j$  and occupation  $o$  at local labor market  $m$  in year  $t$ ,  $s_{i,o|m,t}$  is the employment share of the plant out of the market  $m$ ,  $\log(L_{i,o,t})$  is the log of the establishment-occupation employment,  $\psi_{J(i),o,t}$  is a firm-occupation-year fixed effect,  $\delta_{N(i),t}$  is a commuting zone-year fixed effect and  $\epsilon_{i,o,t}$  is an error term.

There are two potentially endogenous variables,  $s_{i,o|m,t}$  and  $\log(L_{i,o,t})$ , so we follow two approaches. First, we instrument  $s_{i,o|m,t}$  with the presence of mass-layoff shocks in the local labor

market and add the contemporaneous logarithm of employment as a control. Even if this last instrument would not satisfy the standard exclusion restriction, we can still get a consistent estimate of  $\beta$  with a different conditional mean independence assumption. Let  $Z$  be the mass-layoff shock instrument, and  $W$  is the vector of controls, including the log employment and the fixed effects. Then, if  $\mathbb{E}(\epsilon|Z, W) = \mathbb{E}(\epsilon|W) = W\zeta$  we still obtain a consistent estimate of  $\beta$  using the instrument.<sup>6</sup> In the second approach, we use lagged values of the log employment as an instrument instead of its contemporaneous value. Panel (c) of Figure E1 presents the estimates for  $\beta$  estimating the model (E3) using the first approach and Panel (d) the second approach.

## E.2 Labor market concentration and the labor share

We establish the relationship between aggregate concentration measures and the labor share. A standard measure of concentration is the Herfindahl-Hirschman Index (HHI). The HHI of market  $m$  at time  $t$ ,  $HHI_{mt}$ , is the sum of the squared employment shares of the plants present in  $m$  at a given year. The labor share at the 3-digit industry level,  $LS_{ht}$ , is the ratio of the wage bill over value added at time  $t$ . Due to data restrictions of observing value added only at the firm level, we cannot compute labor shares at the local labor market level. We therefore build a sub-industry concentration index  $\overline{HHI}_{ht}$  by taking the employment weighted mean of  $HHI_{mt}$  across different local labor markets.<sup>7</sup> We run the following linear regression:

$$\log(LS_{h,t}) = \delta_{b,t} + \beta \log(\overline{HHI}_{h,t}) + \varepsilon_{h,t}. \quad (\text{E4})$$

Table III3 in the [Supplemental Material](#) complements the results from Table 2 in the main text and confirm that more concentrated sub-industries have a lower labor share.

<sup>6</sup>Proof: Let the regression be  $y = \beta s + W\tilde{\gamma} + \epsilon$ . Assume that  $\mathbb{E}(\epsilon|Z, W) = \mathbb{E}(\epsilon|W) = W\zeta$ . This implies:  $y = \beta s + W\tilde{\gamma} + \epsilon - \mathbb{E}(\epsilon|W) + \mathbb{E}(\epsilon|W) = \beta s + W(\tilde{\gamma} + \zeta) + \tilde{\epsilon}$ , where  $\tilde{\epsilon} = \epsilon - \mathbb{E}(\epsilon|W)$ . Then  $\mathbb{E}(\tilde{\epsilon}|Z, W) = \mathbb{E}(\epsilon|Z, W) - \mathbb{E}(\epsilon|W) = \mathbb{E}(\epsilon|Z, W) - \mathbb{E}(\epsilon|Z, W) = 0$ . Thus, an IV regression gives consistent estimates of  $\beta$ ,  $(\tilde{\gamma} + \zeta)$ .

<sup>7</sup>The HHI at market  $m$  and year  $t$  is:  $HHI_{mt} = \sum_{i \in \mathcal{I}_{m,t}} s_{io|m,t}^2$  where shares at the market are accounted as shares of full time equivalent employees and  $\mathcal{I}_{m,t}$  is the set of all firms in the sub-market  $m$  at year  $t$ . The sub-industry concentration is:  $\overline{HHI}_{ht} = \frac{1}{|\mathcal{M}_{ht}|} \sum_{m \in \mathcal{M}_{ht}} HHI_{mt} \frac{L_{mt}}{L_{ht}}$ , where  $|\mathcal{M}_{ht}|$  is the number of local labor markets that belong to  $h$  in  $t$ ,  $L_{mt}$  is the local labor market employment and  $L_{ht}$  is the 3-digit industry employment.

Table E1: Union Density and Collective Bargaining Coverage

Country	Union Density	Coverage	Country	Union Density	Coverage
<b>Western Europe</b>			<b>Southern Europe</b>		
Austria	27.7	98.0	Italy	36.4	80.0
France	9.0	98.5	Spain	16.8	80.2
Germany	17.7	57.8	<b>Americas</b>		
Netherlands	18.1	85.9	Canada	29.3	30.4
Switzerland	16.1	49.2	Chile	15.3	19.3
<b>Northern Europe</b>			United States	10.7	12.3
Finland	67.6	89.3	<b>Asia &amp; Oceania</b>		
Ireland	26.3	33.5	Australia	15.1	59.9
Norway	49.7	67.0	Japan	17.5	16.9
United Kingdom	25.0	27.5	Korea	10.0	11.9

Notes: Year 2014. Variables in percents. *Union Density*: unionization rate (unionized workers relative to total employment), *Coverage*: collective agreement coverage (ratio of employees covered by collective agreements divided by all wage earners with the right to bargain). OECD data from administrative data except for Australia, Ireland and the United States which are based on survey data. Regions according to the U.N. M49 area codes.

### E.3 Unions

Table E1 presents union density and coverage statistics for several countries.<sup>8</sup> France has the highest coverage.

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<sup>8</sup>OECD data <https://stats.oecd.org/Index.aspx?DataSetCode=TUD>.



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