

Union and Firm Labor Market Power

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This Supplemental Material is organized as follows. Section I presents additional derivations, which includes the establishment labor supply, different extensions of the model and derivations with an alternative production function. Section II presents additional proofs and results on the model aggregation. Section III contains additional estimation results. Section IV contains additional counterfactual. Section V presents additional robustness checks. Section VI presents details on sample selection and variable construction. Section VII presents summary statistics, and Section VIII covers the empirical evidence on rent-sharing.

I Additional derivations

I.1 Establishment-occupation labor supply

To simplify the notation, we get rid of the occupation subscript o in this subsection. The indirect utility of a worker k that is employed in establishment i in sub-market m is: $u_{kim} = w_i z_{i|m}^1 z_m^2$, where $z_{i|m}^1$ and z_m^2 are independent utility shocks. They are both distributed Fréchet with shape and scale parameters ε_b and T_i for $z_{i|m}^1$, and η and 1 for z_m^2 .

Workers first see shocks z_m^2 for all local labor markets. After choosing their labor market, workers then observe the establishment specific shocks. Therefore, there is a two stage decision: first, the worker chooses the local labor market that maximizes her expected utility, and subsequently she chooses the establishment that maximizes her utility conditional on the chosen sub-market.

The unconditional probability of a worker going to establishment i in sub-market m is:

$$\Pi_i = P\left(w_i z_{i|m}^1 \geq \max_{i' \neq i} w_{i'} z_{i'|m}^1\right) P\left(\mathbb{E}_m(\max_i w_i z_{i|m}^1) z_m^2 \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z_{i|m'}^1) z_{m'}^2\right)$$

We first solve for the left term. Let's define the following distribution function:

$$G_i(v) = P\left(w_i z_{i|m}^1 < v\right) = P\left(z_{i|m}^1 < v/w_i\right) = e^{-T_i w_i^{\varepsilon_b} v^{-\varepsilon_b}}.$$

To ease notation, define *conditional utility* $v_i = w_i z_{i|m}^1$ for all i, i' . We need to solve for $P\left(v_i \geq \max_{i' \neq i} v_{i'}\right)$. Fix $v_i = v$. Then we have:

$$P\left(v \geq \max_{i' \neq i} v_{i'}\right) = \bigcap_{i' \neq i} P\left(v_{i'} < v\right) = \prod_{i' \neq i} G_{i'}(v) = e^{-\Phi_m^{-i} v^{-\varepsilon_b}} = G_m^{-i}(v),$$

where $\Phi_m^{-i} \equiv \sum_{i' \neq i} T_{i'} w_{i'}^{\varepsilon_b}$. Similarly, the probability of having at most conditional utility v is: $G_m(v) = P\left(v \geq \max_{i'} v_{i'}\right) = e^{-\Phi_m v^{-\varepsilon_b}}$, where $\Phi_m \equiv \sum_{i'} T_{i'} w_{i'}^{\varepsilon_b}$. Integrating $G_m^{-i}(v)$ over all possible values of v :

$$\begin{aligned} P\left(v_i \geq \max_{i' \neq i} v_{i'}\right) &= \int_0^\infty e^{-\Phi_m^{-i} v^{-\varepsilon_b}} dG_i(v) = \int_0^\infty \varepsilon_b T_i w_i^{\varepsilon_b} v^{-\varepsilon_b-1} e^{-\Phi_m^{-i} v^{-\varepsilon_b}} dv \\ &= \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b-1} e^{-\Phi_m v^{-\varepsilon_b}} dv = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty dG_m(v) = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m}. \end{aligned}$$

Now we need to find $P\left(\mathbb{E}_m(\max_i w_i z_{i|m}^1) z_m^2 \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z_{i|m'}^1) z_{m'}^2\right)$. So, the expected utility of working in sub-market m is:

$$\mathbb{E}_m(\max_i w_i z_{i|m}^1) = \int_0^\infty v_i dG_m(v) = \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b} e^{-\Phi_m v^{-\varepsilon_b}} dv.$$

We define a new variable: $x = \Phi_m v^{-\varepsilon_b}$, $dx = -\varepsilon_b \Phi_m v^{-(\varepsilon_b+1)} dv$. Now we can change variable in the previous integral and obtain:

$$\int_0^\infty x^{-1/\varepsilon_b} \Phi_m^{1/\varepsilon_b} e^{-x} dx = \Gamma\left(\frac{\varepsilon_b - 1}{\varepsilon_b}\right) \Phi_m^{1/\varepsilon_b},$$

where $\Gamma(\cdot)$ is the Gamma function. Defining $\Gamma_b \equiv \Gamma\left(\frac{\varepsilon_b - 1}{\varepsilon_b}\right)$, we can rewrite:

$$P\left(\mathbb{E}_m(\max_i w_i z_{i|m}^1) z_m^2 \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z_{i|m'}^1) z_{m'}^2\right) = P\left(\Phi_m^{1/\varepsilon_b} \Gamma_b z_m^2 \geq \max_{m' \neq m} \Phi_{m'}^{1/\varepsilon_b} \Gamma_b z_{m'}^2\right).$$

Following similar arguments as above, this probability is equal to:

$$P\left(\Phi_m^{1/\varepsilon_b} \Gamma_b z_m^2 \geq \max_{m' \neq m} \Phi_{m'}^{1/\varepsilon_b} \Gamma_b z_{m'}^2\right) = \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi},$$

where $\Phi \equiv \sum_{b' \in \mathcal{B}} \sum_{m' \in \mathcal{M}_{b'}} \Phi_{m'}^{\eta/\varepsilon_{b'}} \Gamma_{b'}^\eta$. Finally, combining the two probabilities we obtain:

$$\Pi_i = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi}.$$

By integrating Π_i over the measure of workers L , the establishment labor supply is: $L_i = \Pi_i \times L$.

Workers' welfare. One way to measure workers welfare would be to compute the average utility for workers. However this is not possible as the estimated shape parameter η is smaller than 1 so the mean for the Fréchet distributed utilities is not defined. Instead, we compute the median utility agents expect to receive in each local labor market. This is equal to:

$$\text{Median} \left[\max_m \mathbb{E}_m (\max_i w_i z_{i|m}^1) z_m^2 \right] \propto \Phi^{1/\eta}.$$

I.2 Extension: Endogenous participation

We incorporate the option of being out-of-the-labor-force (from now on OTLF) by defining a new (3-digit) sub-industry for each (2-digit) sector. These new sub-industries have only one 'establishment', indexed by u , per commuting zone that 'employ' different occupations paying them a home production wage w_{uo} . The establishment-occupations define a new set of local labor markets \mathcal{U} (combinations of commuting zones, occupations, and the new sub-industries).

Similar to the baseline model, we assume that workers face idiosyncratic shocks that have the same Fréchet distributions. The number of workers OTLF in a particular commuting zone-sector u and occupation o is: $L_{uo} = \frac{(T_{uo} w_{uo}^{\varepsilon_b})^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi} L$, $\Phi \equiv \Phi_e + \Phi_u$. L is the total labor supply of employed and OTLF workers. Φ is the aggregate outside option that now formed of two components: Φ_e coming from the outside options of the employed workers and Φ_u from the outside options out of the labor force. We use commuting zone level unemployment rates as proxies for OTLF rates.¹

We assume that the OTLF rate is the same across industries and occupations in each commuting zone and define the proportion of workers OTLF in each local labor market uo accordingly. The proportion of OTLF workers in each local market identifies the home production amenity and income $T_{uo} w_{uo}^{\varepsilon_b}$ which are fixed in the counterfactuals.

¹We lack data on the geographical distribution of OTLF status at the commuting zone. Basing our counterfactuals in those surveys would require the assumption of constant rates of labor participation for entire regions.

We showed in the proof of Proposition 5 that the solution of sector prices \mathbf{P} is homogeneous of degree zero with respect to total employment level which we denote here as L_e . We have that,

$$L_{io}(w_{io}) = \frac{T_{io} w_{io}^{\varepsilon_b} \Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi_m \Phi} L = \frac{T_{io} w_{io}^{\varepsilon_b} \Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi_m \Phi_e} L_e.$$

We have that $L_e = \frac{\Phi_e}{\Phi} L$ with $\Phi_e \equiv \sum_{m \in \mathcal{M}} \Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta$ is the part of Φ that comes from the employed and $\Phi_u \equiv \sum_{u_o \in \mathcal{U}} (T_{u_o} w_{u_o}^{\varepsilon_b})^{\eta/\varepsilon_b} \Gamma_b^\eta$ is the part from the out of the labor force as in the main text.

The model aggregation steps are the same as in Section B with the exception that L_b now is $L_{b,e}$. We normalize all the reservation wages w_{u_o} to 1. We recover the out-of-the-labor-force amenities T_{u_o} to match the observed unemployment rate and we can compute Φ_u . There are no markdowns for the OTLF and we set the productivities of the fictitious OTLF establishments to zero such that they do not contribute to aggregate output.

Aggregating from (A4) from the Online Appendix,

$$\begin{aligned} \Phi_{b,e} &= \left(\frac{\Phi}{L} \right)^{\frac{\eta\delta}{1+\eta\delta}} \sum_{m \in \mathcal{M}_b} \tilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b)}{\varepsilon_b(1+\eta\delta)}} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} = \left(\frac{\Phi}{L} \right)^{\frac{\eta\delta}{1+\eta\delta}} \tilde{\Phi}_{b,e} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \quad (I1) \\ \tilde{\Phi}_{b,e} &\equiv \sum_{m \in \mathcal{M}_b} \tilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b)}{\varepsilon_b(1+\eta\delta)}}, \quad \Phi \equiv \Phi_e + \Phi_u, \end{aligned}$$

and,

$$\Phi_e \equiv \left(\frac{\Phi}{L} \right)^{\frac{\eta\delta}{1+\eta\delta}} \sum_{b \in \mathcal{B}} \tilde{\Phi}_{b,e} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \Gamma_b^\eta = \left(\frac{\Phi}{L} \right)^{\frac{\eta\delta}{1+\eta\delta}} \tilde{\Phi}_e \quad (I2)$$

$$\begin{aligned} \tilde{\Phi}_e &\equiv \sum_{b \in \mathcal{B}} \tilde{\Phi}_{b,e} P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \Gamma_b^\eta \\ L_{b,e} &= \frac{\Phi_{b,e} \Gamma_b^\eta}{\Phi_e} L_e = \frac{\tilde{\Phi}_{b,e} \Gamma_b^\eta P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}}}{\tilde{\Phi}_e} L_e. \quad (I3) \end{aligned}$$

We can solve for the prices without knowing total employment level L_e . Total employment level is $L_e = \frac{\Phi_e}{\Phi} L$, where L is total labor supply (employed and out-of-the-labor-force) that will

determine the level of aggregate output. We can find it by solving for Φ_e in equation (12),

$$\Phi_e^{\frac{1+\eta\delta}{\eta\delta}} L = (\Phi_e + \Phi_u) \tilde{\Phi}_e^{\frac{1+\eta\delta}{\eta\delta}}.$$

The solution is obviously unique as the left hand side is convex and the right hand side linear. With the solution for Φ_e one can construct all the aggregates back.

I.3 Extension: Agglomeration

We assume that the productivity is: $\hat{A}_{io} = \tilde{A}_{io} L_m^{\gamma(1-\alpha_b)}$. The agglomeration effect is a local labor market externality with elasticity $\gamma(1-\alpha_b)$. The wage first order condition is:

$$w_{io} = \beta_b \lambda(\mu_{io}, \varphi_b) Z_{io} L_{io}^{-\delta} L_m^\gamma. \quad (14)$$

Similarly to the baseline counterfactual, we back out the transformed TFPRs, Z_{io} , to match observed establishment-occupation wages, w_{io} , under the assumption of agglomeration externalities. In the case where employment for a given local labor market is high, the backed out productivity of the establishments in that market m is lower than for the main counterfactual.

Plugging the labor supply into (14), the wage in the baseline economy is:

$$w_{io} = \left(\beta_b \lambda(\mu_{io}, \varphi_b) \frac{Z_{io}}{(T_{io} \Gamma_b^\eta)^\delta} \right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi_m^{v_b - \frac{\eta}{\varepsilon_b} \tilde{v}_b} \left(\frac{\Phi}{L} \right)^{\tilde{v}_b}, \quad v_b = \frac{\delta}{1+\varepsilon_b\delta}, \quad \tilde{v}_b = \frac{\delta - \gamma}{1+\varepsilon_b\delta}.$$

The baseline wage can be written as: $w_{io} = \tilde{w}_{io} \Phi_m^{v_b - \frac{\eta}{\varepsilon_b} \tilde{v}_b} \left(\frac{\Phi}{L} \right)^{\tilde{v}_b}$. Analogously, the counterfactual wage is: $w_{io} = \omega_{io} \hat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \Phi_m^{v_b - \frac{\eta}{\varepsilon_b} \tilde{v}_b} \left(\frac{\Phi}{L} \right)^{\tilde{v}_b}$. Aggregating to generate Φ_m ,

$$\Phi_m = \tilde{\Phi}_m^{\frac{1+\varepsilon_b\delta}{1+\eta(\delta-\gamma)}} \left(\frac{\Phi}{L} \right)^{\frac{\varepsilon_b(\delta-\gamma)}{1+\eta(\delta-\gamma)}}. \quad (15)$$

The counterfactual Φ'_m is analogously $\Phi'_m = \tilde{\Phi}'_m^{\frac{1+\varepsilon_b\delta}{1+\eta(\delta-\gamma)}} \hat{P}_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} \left(\frac{\Phi}{L} \right)^{\frac{\varepsilon_b(\delta-\gamma)}{1+\eta(\delta-\gamma)}}$.

In order to be able to find a solution to the model, we need that the exponents are bounded. This is equivalent to requiring $\gamma \neq \frac{1}{\eta} + \delta$. The parameter γ governs the strength of agglomer-

ation forces within a local labor market, and δ and $\frac{1}{\eta}$ are related with dispersion forces. Those come from the decreasing returns to scale (δ) and from the variance of taste shocks ($\frac{1}{\eta}$). When the latter is high, the mass of workers having extreme taste shocks is higher. This implies that agglomeration forces will impact less as workers would be more inelastic to changes in wages. The standard condition for uniqueness of the equilibrium with agglomeration would be that it is sufficiently weak ($\gamma < \frac{1}{\eta} + \delta$). We instead find the weaker condition $\gamma \neq \frac{1}{\eta} + \delta$.

The counterfactual industry labor supply is:

$$L'_b = \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} \widetilde{\Phi}'_b \Gamma_b^\eta}{\sum_{b \in \mathcal{B}} \widehat{P}_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta(\delta-\gamma))}} \widetilde{\Phi}'_{b'} \Gamma_{b'}^\eta}, \quad \widetilde{\Phi}'_b \equiv \sum_{m \in \mathcal{M}_b} \widetilde{\Phi}'_m^{\frac{\eta(1+\varepsilon_b \delta)}{\varepsilon_b(1+\eta(\delta-\gamma))}}$$

The counterfactual establishment-occupation output y'_{io} and sector output Y'_b are:

$$y'_{io} = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} Z_{io} L'_{io}{}^{1-\delta} L'_m{}^\gamma, \quad Y'_b = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} \bar{Z}_b \bar{\Omega}'_b L'_b{}^{1-\delta+\gamma},$$

where γ changed the returns to scale and the aggregations. We define:

$$\bar{\Omega}'_b \equiv \sum_{io \in \mathcal{I}_b} \frac{A_{io}}{A_b} s'_{io|m}{}^{1-\delta} s'_{m|b}{}^{1-\delta+\gamma}$$

$$\bar{Z}_b \equiv \sum_{io \in \mathcal{I}_b} Z_{io} \bar{s}_{io|m}{}^{1-\delta} \bar{s}_{m|b}{}^{1-\delta+\gamma},$$

where \bar{Z}_b is a measure of sector productivity under the efficient allocation with agglomeration (that is the same in the baseline and in the counterfactual) and $\bar{\Omega}'_b$ is a measure of misallocation in the counterfactual.

The expressions for the baseline are analogous but setting $\widehat{P}_b = 1$, and defining the above with baseline employment shares, $Y'_b = \frac{1}{P_b} \bar{Z}_b \bar{\Omega}_b L_b{}^{1-\delta+\gamma}$.

The intermediate good demand in the counterfactual relative to the baseline is:

$$\begin{aligned} \widehat{P}_b^{\frac{1}{1-\alpha_b}} \widehat{\Omega}_b \left(\frac{L'_b(\widehat{\mathbf{P}})}{L_b} \right)^{1-\delta+\gamma} &= \prod_{b' \in \mathcal{B}} \widehat{P}_b^{\frac{\alpha_{b'}}{1-\alpha_{b'}}} \widehat{\Omega}_{b'} \left(\frac{L'_{b'}(\widehat{\mathbf{P}})}{L_{b'}} \right)^{1-\delta+\gamma} \\ \Leftrightarrow \widehat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta(\delta-\gamma))}} \widehat{\Omega}_b \left(\frac{\widetilde{\Phi}'_b \Gamma_b^\eta}{L_b} \right)^{1-\delta+\gamma} &= \prod_{b' \in \mathcal{B}} \widehat{P}_{b'}^{\alpha_{b'}(1+\eta(\delta-\gamma))+\eta(1-\delta+\gamma)} \widehat{\Omega}_{b'} \left(\frac{\widetilde{\Phi}'_{b'} \Gamma_{b'}^\eta}{L_{b'}} \right)^{1-\delta+\gamma}. \end{aligned}$$

Uniqueness of the solution to this system of equations is guaranteed by $\sum_{b \in \mathcal{B}} \alpha_b \theta_b < 1$. This condition being the same as for Proposition 5, uniqueness of the equilibrium with agglomeration forces only needs the additional requirement of $\gamma \neq \frac{1}{\eta} + \delta$.

I.4 Alternative production function

For the interested reader, in this section we lay out a model with an alternative Cobb-Douglas production function with generic capital and a labor composite that is at odds with the data.

Suppose that establishment i produces using some generic capital K_i and a labor composite H_i of different occupations:

$$y_i = \widetilde{A}_i K_i^{\alpha_b} H_i^{\beta_b} = \widetilde{A}_i K_i^{\alpha_b} \left(\prod_{o \in \mathcal{O}} L_{io}^{\gamma_o} \right)^{\beta_b}, \quad \sum_o \gamma_o = 1, \quad \alpha_b + \beta_b \leq 1. \quad (\text{I6})$$

The first order conditions with respect to capital and the bargained wage are:

$$\begin{aligned} w_{io} &= \beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) P_b \frac{y_i}{L_{io}}, \\ R_b &= \alpha_b \widetilde{A}_i K_i^{\alpha_b - 1} H_i^{\beta_b}. \end{aligned}$$

Substituting the first order condition for capital into the production function, the wage first order condition becomes:

$$w_{io} = \beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) A_i H_i^{1-\delta} L_{io}^{-1} P_b^{\frac{1}{1-\alpha_b}},$$

where we plugged the labor supply and used the definition of $\delta = 1 - \frac{\beta_b}{1-\alpha_b}$ from the main text

and $A_i = \tilde{A}_i^{\frac{1}{1-\alpha_b}} \left(\frac{\alpha_b}{R_b} \right)^{\frac{\alpha_b}{1-\alpha_b}}$ as in the main text. Using those and solving for L_{io} , we can write the labor composite H_i as function of wages:

$$H_i^\delta = P_b^{\frac{1}{1-\alpha_b}} \prod_{o \in \mathcal{O}} \beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) w_{io}^{-1}$$

Substituting the wage equation with the labor supply (7) into the expression above, we get:

$$\begin{aligned} H_i^{1+\varepsilon_b \delta} &= P_b^{\frac{\varepsilon_b}{1-\alpha_b}} \prod_{o \in \mathcal{O}} \left(\beta_b \gamma_o \lambda(\mu_{io}, \varphi_b) A_i (T_{io} \Gamma_b^\eta)^{1/\varepsilon_b} \right)^{\varepsilon_b \gamma_o} \prod_{o \in \mathcal{O}} \left(\Phi_m^{1-\eta/\varepsilon_b} \frac{\Phi}{L} \right)^{-\gamma_o} \\ &= P_b^{\frac{\varepsilon_b}{1-\alpha_b}} (\beta_b Y A_i)^{\varepsilon_b} T_i \Gamma \prod_{o \in \mathcal{O}} \lambda(\mu_{io}, \varphi_b)^{\varepsilon_b \gamma_o} \prod_{o \in \mathcal{O}} \left(\Phi_m^{1-\eta/\varepsilon_b} \frac{\Phi}{L} \right)^{-\gamma_o}, \end{aligned}$$

where $Y \equiv \prod_{o \in \mathcal{O}} \gamma_o$, $\Gamma \equiv \prod_{o \in \mathcal{O}} \Gamma_b^\eta$ and $T_i \equiv \prod_{o \in \mathcal{O}} T_{io}$. Plugging back into the wage equation and rearranging, we get:

$$\begin{aligned} w_{io} &= \left[\lambda(\mu_{io}, \varphi_b) \frac{\gamma_o}{T_{io} \Gamma_b^\eta} (\beta_b A_i)^{\frac{1+\varepsilon_b}{1+\varepsilon_b \delta}} (Y (T_i \Gamma)^{1/\varepsilon_b})^{\frac{\varepsilon_b(1-\delta)}{1+\varepsilon_b \delta}} \right. \\ &\quad \times \left. \left(\prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma_{o'}} \right)^{\frac{1-\delta}{1+\varepsilon_b \delta}} \left(\prod_{o' \in \mathcal{O}} \Phi_{m'}^{(\eta/\varepsilon_b-1)\gamma_{o'}} \right)^{\frac{1-\delta}{1+\varepsilon_b \delta}} \Phi_m^{1-\eta/\varepsilon_b} \right]^{\frac{1}{1+\varepsilon_b}} \left(\frac{\Phi}{L} \right)^{\frac{1}{1+\varepsilon_b}} P_b^{1/\chi_b}, \quad (I7) \end{aligned}$$

with $\chi_b = (1 - \alpha_b)(1 + \varepsilon_b \delta)$. Define the following:

$$\begin{aligned} c_{io} &\equiv \frac{\gamma_o}{T_{io} \Gamma_b^\eta} (\beta_b A_i)^{\frac{1+\varepsilon_b}{1+\varepsilon_b \delta}} (Y (T_i \Gamma)^{1/\varepsilon_b})^{\frac{\varepsilon_b(1-\delta)}{1+\varepsilon_b \delta}}, \\ C_l &\equiv \prod_{o' \in \mathcal{O}} \left(\Phi_{m'}^{(\eta/\varepsilon_b-1)\gamma_{o'}} \right)^{\frac{\delta}{1+\varepsilon_b \delta}} \left(\frac{\Phi}{L} \right)^{\frac{1}{1+\varepsilon_b}}, \\ F_b &\equiv P_b^{1/\chi_b}, \end{aligned}$$

where C_l is a location constant with $l = n \times h$. Rearranging we have that:

$$w_{io} = \left[\lambda(\mu_{io}, \varphi_b) c_{io} \left(\prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma_{o'}} \right)^{\frac{1-\delta}{1+\varepsilon_b \delta}} \frac{\Phi_m^{1-\eta/\varepsilon_b}}{\prod_{o' \in \mathcal{O}} \Phi_{m'}^{(1-\eta/\varepsilon_b)\gamma_{o'}}} \right]^{\frac{1}{1+\varepsilon_b}} C_l F_b. \quad (I8)$$

The last system is equivalent to the one in (I7) and has the benefit to being able to write the wages as $w_{io} = \tilde{w}_{io} C_m F_b$, where we want \tilde{w}_{io} to be homogeneous of degree zero with respect constants to m level. Note that the last term inside the brackets is homogeneous of degree zero with respect to location l constants shared by all the occupations of a establishments. Then, defining $\tilde{\Phi}_m \equiv \sum_{i \in \mathcal{I}_m} T_{io} w_{io}^{\varepsilon_b}$, the establishment-occupation or normalized wage is:

$$\tilde{w}_{io} \equiv \left[\lambda(\mu_{io}, \varphi_b) c_{io} \left(\prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma_o'} \right)^{\frac{1-\delta}{1+\varepsilon_b \delta}} \frac{\tilde{\Phi}_m^{1-\eta/\varepsilon_b}}{\prod_{o' \in \mathcal{O}} \tilde{\Phi}_{m'}^{(1-\eta/\varepsilon_b) \gamma_o'}} \right]^{\frac{1}{1+\varepsilon_b}}. \quad (\text{I9})$$

\tilde{w}_{io} is homogeneous of degree zero with respect to location l constants shared by all occupations. This property, makes the model with the alternative production function also block recursive. That is, it allows solving for the normalized wages of every location l (combinations of commuting zone n and sub-industry h combinations) independently and then recover the aggregate constants. Aggregating (I9) and solving for $\tilde{\Phi}_m$, we have:

$$\tilde{\Phi}_m = \left[\frac{\sum_{i \in \mathcal{I}_m} \left(\lambda(\mu_{io}, \varphi_b) c_{io} T_{io}^{\frac{1+\varepsilon_b}{\varepsilon_b}} \prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma_o'} \right)^{\frac{1-\delta}{1+\varepsilon_b \delta}}}{\prod_{o' \in \mathcal{O}} \tilde{\Phi}_{m'}^{(1-\eta/\varepsilon_b) \gamma_o'}} \right]^{\frac{\varepsilon_b}{1+\eta}}.$$

Taking everything to the power $(1 - \eta/\varepsilon_b) \gamma_o$ and taking the product,

$$\mathcal{L}_l \equiv \prod_{o' \in \mathcal{O}} \tilde{\Phi}_{m'}^{(1-\eta/\varepsilon_b) \gamma_o'} = \prod_{o' \in \mathcal{O}} \left[\sum_{i \in \mathcal{I}_m} \left(\lambda(\mu_{io}, \varphi_b) c_{io} T_{io}^{\frac{1+\varepsilon_b}{\varepsilon_b}} \prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma_o'} \right)^{\frac{1-\delta}{1+\varepsilon_b \delta}} \right]^{\gamma_{o'} \frac{\varepsilon_b - \eta}{1+\varepsilon_b - \eta}},$$

which recovers all the local labor market constants inside \tilde{w}_{io} .

In order to prove the existence and uniqueness of the solution of the system (I9), define \hat{w}_{io}

as:

$$\begin{aligned}\widehat{w}_{io} &= \left[\lambda(\mu_{io}, \varphi_b) \left(\prod_{o' \in \mathcal{O}} \lambda(\mu_{io'}, \varphi_b)^{\varepsilon_b \gamma'_{o'}} \right)^{\frac{1-\delta}{1+\varepsilon_b \delta}} \right]^{\frac{1}{1+\varepsilon_b}} c_{io}^{\frac{1}{1+\varepsilon_b}} \\ w_{io} &= \widehat{w}_{io} \left[\frac{\widetilde{\Phi}_m^{1-\eta/\varepsilon_b}}{\mathcal{L}_l} \right]^{\frac{1}{1+\varepsilon_b}} C_l F_b = \widehat{w}_{io} z_l = \widetilde{w}_{io} C_l F_b.\end{aligned}\quad (\text{I10})$$

We can show that the system formed by (I10) has a solution and is unique.

Proposition 5. For given parameters $0 \leq \alpha_b, \beta_b < 1$, $1 < \eta < \varepsilon_b$, $0 \leq \delta \leq 1$, transformed price F_b , constants C_l , $\widetilde{\Phi}_m$, \mathcal{L}_l and non-negative vectors of productivities $\{A_i\}_{i \in m}$ and amenities $\{T_{io}\}_{io \in m}$, there exists a unique vector of wages $\{w_{io}\}_{io \in I_m}$ for every location l (combination of commuting zone n and sub-industry h) that solves the system formed by (I10).

Proof. For existence, first note that $\lambda(\mu_{io}, \varphi_b) \in \left[(1 - \varphi_b) \frac{\eta}{1+\eta} + \varphi_b \frac{1}{1-\delta}, (1 - \varphi_b) \frac{\varepsilon_b}{1+\varepsilon_b} + \varphi_b \frac{1}{1-\delta} \right], \forall i, o$. Define a vector \mathbf{w} with wage of all the establishment-occupations at location l , $\mathbf{w} \equiv \{w_{11}, w_{12}, \dots, w_{1O}, \dots, w_{I1}, \dots, w_{IO}\}$. Taking for now the elements of z_l as constants. The system to solve is: $f_{io}(\mathbf{w}) = \widehat{w}_{io} z_l$. We have that

$$\begin{aligned}\mathbf{w} \in \mathcal{C} &\equiv \left[\left((1 - \varphi_b) \frac{\eta}{1+\eta} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{11}^{\frac{1}{1+\varepsilon_b}} z_{l1}, \left((1 - \varphi_b) \frac{\varepsilon_b}{1+\varepsilon_b} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{11}^{\frac{1}{1+\varepsilon_b}} z_{l1} \right] \\ &\times \dots \times \left[\left((1 - \varphi_b) \frac{\eta}{1+\eta} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{IO}^{\frac{1}{1+\varepsilon_b}} z_{lO}, \left((1 - \varphi_b) \frac{\varepsilon_b}{1+\varepsilon_b} + \varphi_b \frac{1}{1-\delta} \right)^{\frac{1}{1+\eta\delta}} c_{IO}^{\frac{1}{1+\varepsilon_b}} z_{lO} \right].\end{aligned}$$

The system f_{io} is continuous on wages and maps into itself on \mathcal{C} . The last set being a compact set we can apply Brouwer's fixed point theorem.

For uniqueness, once the product of the wedges is substituted, \widehat{w}_{io} is:

$$\widehat{w}_{io} = \left[\lambda(\mu_{io}, \varphi_b) c_{io} \prod_{o' \in \mathcal{O}} (w_{io'} c_{io'}^{-\frac{1}{1+\varepsilon_b}})^{\gamma'_{o'} \varepsilon_b (1-\delta)} \right]^{\frac{1}{1+\varepsilon_b}}$$

Define the function $g_{io}(\mathbf{w}) = f_{io}(\mathbf{w}) - w_{io}$. Gross substitution is fulfilled if $\frac{\partial g_{io}(\mathbf{w})}{\partial w_{io}} > 0, \forall j \neq i$ with $j \in \mathcal{I}_l$ and $\frac{\partial g_{io}(\mathbf{w})}{\partial w_{io'}} > 0, \forall o'$. Gross substitution resumes to taking the partial derivatives of \widehat{w}_{io} which are positive by similar reasoning as in the main proof. Finally, \widehat{w}_{io} is homogeneous of

degree $\frac{\varepsilon_b}{1+\varepsilon_b}(1-\delta) < 1$. Therefore the solution to the system (I10) exists and is unique. \square

Finally, the model can be aggregated up to the sector following similar steps as in Proposition 4.

II Additional results and proofs

We use the following Theorem and Corollary to establish uniqueness in our proofs. These are taken from Allen, Arkolakis, and Li (2016) as they are not present any more in the current version of their paper Allen, Arkolakis, and Li (2023). Of course, any error should be attributed to us.

Theorem 1. Consider $g : \mathbb{R}_{++}^n \times \mathbb{R}_{++}^m$ for some $n \in \{1, \dots, N\}$ and $m \in \{1, \dots, M\}$ such that:

1. homogeneity of any degree: $g(tx, ty) = t^k g(x, y)$, $t \in \mathbb{R}_{++}$ and $k \in \mathbb{R}$,
2. gross-substitution property: $\frac{\partial g_i}{\partial x_j} > 0$ for all $i \neq j$,
3. monotonicity with respect to the joint variable: $\frac{\partial g_i}{\partial y_k} \geq 0$, for all i, k .

Then, for any given $y^0 \in \mathbb{R}_{++}^M$ there exists at most one solution satisfying $g(x, y^0) = 0$.

Proof. We proceed by contradiction. Suppose there are two different up-to-scale, solutions, x^1 , x^2 , such that $f(x^1) = f(x^2) = 0$ i.e. $g(x^1, y^0) = g(x^2, y^0) = 0$. Without loss of generality, suppose there exists some $t > 1$ such that $tx_j^1 \geq x_j^2$ for all $j \in \{1, \dots, n\}$ and the equality holds for at least one $j = \bar{j}$. Then the inequality must strictly hold since x^1 and x^2 are different up-to-scale. Condition (iii) $\frac{\partial g_i}{\partial y_k} \geq 0$, for all i, k implies that $g(tx^1, y^0) \leq g(tx^1, ty^0) = 0$ where $g(tx^1, ty^0) = 0$ is from condition (i) (and also $g(tx^2, ty^0) = 0$ because x^1 and x^2 are solutions). However, condition (ii) implies $g_j(tx^1, y^0) > g_j(x^2, y^0) = 0$, thus a contradiction. \square

Corollary 1. Assume (i) $f(x)$ satisfies gross-substitution and (ii) $f(x)$ can be decomposed as $f(x) = \sum_{j=1}^{v_f} g^j(x) - \sum_{k=1}^{v_g} h^k(x)$, where $g^j(x), h^k(x)$ are non-negative vector functions and, respectively, homogeneous of degree α_j and β_k , with $\bar{\alpha} = \max \alpha_j \leq \min \beta_k$.

1. Then there is at most one up-to-scale solution of $f(x) = 0$.

2. In particular, if for some j, k $\alpha_j \neq \beta_k$, then there is at most one solution.

Proof. Define $m(x, y)$ as a vector function where $m_i(x, y) = \sum_{j=1}^{V_f} y^{\bar{\alpha}-\alpha_j} g_i^j(x) - \sum_{k=1}^{V_g} y^{\bar{\alpha}-\beta_k} h_i^k(x)$. Obviously, $m(x, y)$ is of homogenous degree $\bar{\alpha}$ and $\frac{\partial m_i}{\partial y} \geq 0$. Also we have $f(x) = m(x, y^0)$ where $y^0 = 1$, thus the above theorem applies.

Furthermore, if $f_i(x)$ is not homogeneous of some degree because $\alpha_j \neq \beta_k$, there is at most one solution. Suppose not, if tx^1 and x^1 are the solutions, then $f_i(x^1) > t^{-\min(\beta_k)} f_i(tx^1) = 0$, also a contradiction. \square

II.1 Existence and uniqueness of local market equilibrium in Berger et al. (2022)

Our existence and uniqueness proof extends easily to the case consider by Berger et al. (2022), where instead of using shares of employment $s_{io|m}$, they use wage bill shares $s_{io|m}^w = \frac{w_{io} L_{io}}{\sum_{j \in \mathcal{I}_m} w_{jo} L_{jo}}$, and no bargaining power. i.e. $\varphi_b = 0$. The existence proof is exactly the same. For uniqueness and to establish gross-substitution of a similar function $g_{io}(\mathbf{w})$, we can follow all the steps of the previous proof and note that:

$$s_{io|m}^w = \frac{T_{io} w_{io}^{1+\varepsilon_b}}{\sum_{j \in \mathcal{I}_m} T_{jo} w_{jo}^{1+\varepsilon_b}}.$$

Thus, clearly, $\frac{\partial s_{io|m}}{\partial w_{jo}} < 0$ for any $i \neq j$ and $g_{io}(\mathbf{w})$ also satisfies the gross-substitution property. Then we can conclude that the local labor market equilibrium of Berger et al. (2022) also exists and is unique.

II.2 Aggregation of the model

Here we provide additional details that correspond to the aggregation of the model. Parts of these were already covered in the proof of Proposition 4 in the Online Appendix. We include them in the interest of clarity.

Establishment decision. In the absence of bargaining, establishments post a wage and choose capital quantity to maximize profits subject to their individual labor supply taking only into ac-

count the effect on their local labor market. Establishments act strategically. They look at probability Π_{io} and take into account the effect of wages on the establishment-occupation term $T_{io}w_{io}^{\varepsilon_b}$ and also on the local labor market aggregate Φ_m . However, they take as given economy-wide aggregates (Φ and L). This can happen because of a myopic behavior from the establishments or if there is a continuum of local labor markets. The profit maximization problem is:

$$\max_{w_{io}, K_{io}} P_b \tilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b} - w_{io} L_{io}(w_{io}) - R_b K_{io}, \quad (\text{II1})$$

where $L_{io}(w_{io})$ is the labor supply (7). They take Φ and L as given but internalize their effect on Φ_m . P_b and R_b are respectively the sector price and rental rate of capital. The first order conditions are:

$$w_{io} = \beta_b \frac{e_{io}}{e_{io} + 1} P_b \tilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b - 1} \quad R_b = \alpha_b P_b \tilde{A}_{io} K_{io}^{\alpha_b - 1} L_{io}^{\beta_b}. \quad (\text{II2})$$

$e_{io} = \varepsilon_b (1 - s_{io|m}) + \eta s_{io|m}$ is the elasticity of supply for establishment i in occupation o .

When the establishments solve their wage posting problem they act strategically. They look at probability Π_{io} and take into account the effect of wages on the establishment-occupation term $T_{io}w_{io}^{\varepsilon_b}$ and also on the local labor market aggregate Φ_m . However, they take as given economy-wide aggregates (Φ and L).² The finite set of establishments per local labor market generates strategic interactions among the competitors. The strategic interactions within a local labor market induces oligopsonistic competition that features a heterogeneous markdown. The first order condition for the establishment-occupation wage under oligopsonistic competition is: $w_{io}^O = \frac{e_{io}(s_{io|m})}{e_{io}(s_{io|m})+1} \beta_b A_{io} L_{io}^{\frac{\beta_b}{1-\alpha_b}-1} P_b^{\frac{1}{1-\alpha_b}}$, where $e_{io}(s_{io|m}) = \varepsilon_b (1 - s_{io|m}) + \eta s_{io|m}$ is the labor supply elasticity that depends on employment shares.³ This expression is similar to [Card et al. \(2018\)](#) with the difference that we have variable elasticities that arise from the strategic inter-

²Similar to [Atkeson and Burstein \(2008\)](#), this type of behavior could be rationalized either by assuming a myopic behavior of the establishment or by having a continuum of local labor markets.

³The labor supply elasticity in [Berger et al. \(2022\)](#) is related to payroll shares. This difference comes from the fact that agents in their model make an intensive labor supply decision (equation (B3) in their online appendix) while in ours they do not (which would be equivalent to labor supply being their equation on top of (B3) times employment). Under Bertrand competition, the labor supply elasticity in their model is: $\frac{\partial \log(L_{io})}{\partial \log(w_{io})} = \eta + (\theta - \eta) \frac{\partial \log(W_j)}{\partial \log(w_{ij})}$. The latter partial derivative in their framework is $(\frac{w_{ij}}{W_j})^{1+\eta}$ which is the payroll share of the establishments. Note that if one was abstracting from the intensive labor supply margin, that wage ratio would be equal to the employment share as it can be seen in the equation on top of (B3) in their online appendix.

action between establishments. The fraction $\frac{e_{io}(s_{io|m})}{e_{io}(s_{io|m})+1}$ is the markdown from Proposition 1. In the absence of bargaining, the wedge between the marginal revenue product of labor and the wages is a markdown (8) which we denote in short as μ_{io} .

As long as workers find different local labor markets to be less substitutable than establishments within a local labor market (i.e. as long as $\eta < \varepsilon_b$), the markdown (8) is a decreasing function of the employment share $s_{io|m}$. Once an establishment is big with respect to the nearby competitors, it internalizes that it is facing a more inelastic labor supply of workers willing to stay and applies a markdown further away from 1. In the limit where ε_b or η tend to infinity, establishments face an infinitely elastic labor supply and the labor market would be perfectly competitive with a markdown $\mu(s_{io|m}) = 1$.

Heterogeneous markdowns distort relative wages across establishment-occupations and therefore the labor allocations which might generate misallocation of resources and potentially reduce aggregate output even in the case where total employment is fixed.

Abstracting from capital, when the markdowns are constant and total labor supply fixed, labor market power has no effect on output and only affects the division of output into the labor and profit shares. This is no longer true if we were to allow an endogenous leisure or labor force participation decision. Counterfactually increasing wages would increase total labor supply L and therefore total output.⁴

We can use the first order conditions of capital to substitute it into the establishment's production function and obtain an expression that depends only in labor:

$$y_{io} = \left(\frac{\alpha_b}{R_b} \right)^{\frac{\alpha_b}{1-\alpha_b}} \tilde{A}_{io}^{\frac{1}{1-\alpha_b}} L_{io}^{\frac{\beta_b}{1-\alpha_b}} P_b^{\frac{\alpha_b}{1-\alpha_b}}. \quad (\text{II3})$$

To gain tractability in the solution of the model, we restrict the output elasticity with respect to capital, such that $1 - \frac{\beta_b}{1-\alpha_b} = \delta$, where $\delta \in [0, 1]$ is a constant across sectors. This specification nests a constant returns to scale technology when $\delta = 0$. As long as $0 < \delta < 1$ the establishment faces decreasing returns to scale within occupations. Define a transformed productivity

⁴The constant $\mu = \frac{\eta}{\eta+1}$ drives down the wages. If total labor supply were endogenous, workers' decision between consumption c and leisure l would be distorted. Denote by w the wage under monopsonistic competition and by \tilde{w} the wage under a competitive labor market. Worker's maximization under endogenous labor supply leads the marginal rate of substitution to be equal to the wage rate. $w < \tilde{w}$ and therefore $-\frac{U_l}{U_c} = w < \tilde{w}$. Meaning that workers would supply less labor than in the perfectly competitive case.

$A_{io} \equiv \tilde{A}_{io}^{\frac{1}{1-\alpha_b}} \left(\frac{\alpha_b}{R_b}\right)^{\frac{\alpha_b}{1-\alpha_b}}$. The establishment-occupation production is: $y_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} L_{io}^{1-\delta}$. Maximization (III1) is equivalent to:

$$\max_{w_{io}} (1 - \alpha_b) P_b^{\frac{1}{1-\alpha_b}} A_{io} L_{io}^{1-\delta} - w_{io} L_{io}(w_{io}), \quad (\text{II4})$$

Aggregation. Aggregating establishment-occupation output (6) and using the restriction $\frac{\beta_b}{1-\alpha_b} = 1 - \delta \in [0, 1]$, the local labor market output is:

$$Y_m = \sum_{i \in \mathcal{I}_m} y_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} L_{io}^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} s_{io|m}^{1-\delta} L_m^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_m A_m L_m^{1-\delta},$$

where the local labor market productivity and misallocation are measured as:

$$\Omega_m \equiv \sum_{i \in \mathcal{I}_m} \frac{A_{io}}{A_m} s_{io|m}^{1-\delta}, \quad A_m \equiv \sum_{i \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta}, \quad \tilde{s}_{io|m} = \frac{\left(T_{io}^{1/\varepsilon_b} A_{io}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}.$$

The definition of $\tilde{s}_{io|m}$ comes from Proposition 2 with constant labor wedges.

Further aggregating to sector level according to (4):

$$Y_b = \sum_{m \in \mathcal{M}_b} Y_m = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{m \in \mathcal{M}_b} \Omega_m A_m L_m^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_b A_b L_b^{1-\delta}. \quad (\text{II5})$$

The sector level measures of productivity and misallocation are:

$$\begin{aligned} \Omega_b &\equiv \sum_{m \in \mathcal{M}_b} \Omega_m \frac{A_m}{A_b} s_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{io \in \mathcal{I}_m} \frac{A_{io}}{A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}, \\ A_b &\equiv \sum_{m \in \mathcal{M}_b} A_m \tilde{s}_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{io \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta}, \\ \tilde{s}_{m|b} &= \frac{\left[\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} \right]^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta)}}}{\sum_{m' \in \mathcal{M}_b} \left[\sum_{j' \in \mathcal{I}_{m'}} \left(T_{j'o}^{1/\varepsilon_b} A_{j'o}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} \right]^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta)}}}. \end{aligned}$$

A_b is an employment weighted industry productivity with the employment shares that would arise with constant labor wedges. Similarly, Ω_b is an employment weighted sum of productivities where employment shares incorporate the labor wedge normalized by A_b . The covariance between productivities and employment shares is key in order to determine sector productivity. As long as market power distorts the employment distribution making more productive firms to constrain their size, the covariance between productivity and employment is lower than in the case with constant wedges.

Turning to wages, from (9), the establishment wage bill is:

$$w_{io}L_{io} = \beta_b P_b^{\frac{1}{1-\alpha_b}} \lambda_{io} A_{io} L_{io}^{1-\delta} = \beta_b \lambda_{io} P_b y_{io},$$

where we used the production function (6). The local labor market wage bill is,

$$\begin{aligned} \sum_{i \in \mathcal{I}_m} w_{io} L_{io} &= \beta_b \sum_{i \in \mathcal{I}_m} \lambda_{io} P_b y_{io} = \beta_b \sum_{i \in \mathcal{I}_m} \lambda_{io} \frac{P_b y_{io}}{P_b Y_m} P_b Y_m = \beta_b \lambda_m P_b Y_m, \\ \lambda_m &\equiv \sum_{i \in \mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_m A_m} s_{io|m}^{1-\delta}, \end{aligned}$$

where λ_m is a value added weighted sum of establishment labor wedges. Aggregating to the sector,

$$\begin{aligned} \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} w_{io} L_{io} &= \beta_b \sum_{m \in \mathcal{M}_b} \lambda_m \frac{P_b Y_m}{P_b Y_b} P_b Y_b = \beta_b \lambda_b P_b Y_b, \\ \lambda_b &\equiv \sum_{m \in \mathcal{M}_b} \lambda_m \frac{A_m \Omega_m}{\Omega_b A_b} s_{m|b}^{1-\delta} = \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_b A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}. \end{aligned}$$

Using the sectoral production function (II5) and the final good production function (2) we have that:

$$\begin{aligned} Y &= \prod_{b \in \mathcal{B}} \left(P_b^{\frac{\alpha_b}{1-\alpha_b}} A_b \Omega_b L_b^{1-\delta} \right)^{\theta_b} = \prod_{b \in \mathcal{B}} P_b^{\frac{\alpha_b \theta_b}{1-\alpha_b}} \prod_{b \in \mathcal{B}} \left(A_b \Omega_b s_b^{1-\delta} \right)^{\theta_b} L^{1-\delta} \\ &= \prod_{b \in \mathcal{B}} \bar{P}_b^{\frac{\alpha_b \theta_b}{1-\alpha_b}} \prod_{b \in \mathcal{B}} \left[\Omega_b \frac{A_b}{A} s_b^{1-\delta} \left(\frac{P_b}{\bar{P}_b} \right)^{\frac{\alpha_b}{1-\alpha_b}} \right]^{\theta_b} A L^{1-\delta} = \bar{P} \Omega A L^{1-\delta}, \end{aligned}$$

where:

$$\begin{aligned}\bar{P} &\equiv \prod_{b \in \mathcal{B}} \bar{P}_b^{\frac{\alpha_b \theta_b}{1-\alpha_b}}, & \Omega &\equiv \prod_{b \in \mathcal{B}} \left[\Omega_b \frac{A_b}{A} s_b^{1-\delta} \left(\frac{P_b}{\bar{P}_b} \right)^{\frac{\alpha_b}{1-\alpha_b}} \right]^{\theta_b} \\ A &\equiv \sum_{b \in \mathcal{B}} \sum_{m \in \mathcal{M}_b} \sum_{io \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta} \tilde{s}_b^{1-\delta} = \sum_{b \in \mathcal{B}} A_b \tilde{s}_b^{1-\delta} \\ \tilde{s}_b &= \frac{\sum_{m \in \mathcal{M}_b} \left[\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}} \right]^{\frac{\eta(1+\varepsilon_b \delta)}{\varepsilon_b(1+\eta)}}}{\sum_{b' \in \mathcal{B}} \sum_{m' \in \mathcal{M}_{b'}} \left[\sum_{j' \in \mathcal{I}_{m'}} \left(T_{j'o}^{1/\varepsilon_{b'}} A_{j'o} \right)^{\frac{\varepsilon_{b'}}{1+\varepsilon_{b'} \delta}} \right]^{\frac{\eta(1+\varepsilon_{b'} \delta)}{\varepsilon_{b'}(1+\eta)}}}.\end{aligned}$$

Ω represents an aggregate misallocation measure taking into account general equilibrium effects, \bar{P}_b is the price of sector b good if all the labor wedges in the economy were constant and A is a measure of undistorted productivity.

Aggregate labor share. From the above, the sector labor share is,

$$LS_b = \beta_b \lambda_b, \quad LS = \frac{\sum_{b \in \mathcal{B}} \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} w_{io} L_{io}}{\sum_{b \in \mathcal{B}} \sum_{m \in \mathcal{M}_b} \sum_{i \in \mathcal{I}_m} P_b Y_{io}}. \quad (\text{II6})$$

Realizing that industry b expenditure share is equal to θ_b , the aggregate labor share is $LS = \sum_{b \in \mathcal{B}} \beta_b \lambda_b \theta_b$. For given parameters, knowing the industry wedges $\{\lambda_b\}_{b=1}^B$ is enough to compute the aggregate labor share.

III Additional estimation results

In Table III2 we provide additional estimation results to complement the ones from the Online Appendix ???. We compare the baseline estimates of the within local labor market elasticities of substitution to estimates with two-period lagged instruments in Table III2 and complement the aggregate estimates from Table 2 in Table III3.

Sector fixed effects in Table III3 capture differences in the usage of capital. The focus of the paper being the cross sectional allocation of resources we also take sector-year fixed effects to

use only cross sectional variation.⁵ Column 3 shows that the negative relation between employment concentration and the labor share is robust to controlling for sector and sector-year fixed effects. This regression gives a sense of the importance of the labor wedge heterogeneity to generate output and labor share losses. At face value, the estimate with sector fixed effects (column 2) implies a reduction of 1 percentage point of the labor share when passing from the first to the third quartile of concentration.⁶ Estimates in column 3 with sector-year fixed effects are very similar. The low estimated effects imply that wages, and therefore labor shares, are not very responsive to differentiated levels of concentration. Nevertheless, one cannot interpret that they rule out employer labor market power because in a setting where all the firms acted as pure monopsonists facing an equal labor supply elasticity, wages (and the labor share) would be insensitive to concentration as all establishments would have the same markdown. The small estimated coefficient is most likely a result of level effects as the regression does not take into account the effect of concentration on the average level of the labor share as this is absorbed by the fixed effects.

Sector estimates

Table III1 presents the estimated output elasticities with respect to labor, within industry elasticities, the workers' bargaining power for every 2-digit industry, the elasticities of the final good production function and the rental rates of capital for 2007.

We calibrate the elasticities of the final good production function $\{\theta_b\}_{b \in \mathcal{B}}$ yearly such that the industry expenditure shares are equal to the shares of industry value added in the data. Table III1 has the calibrated final good production function elasticities of the intermediate good $\{\theta_b\}_{b=1}^{\mathcal{B}}$ and the rental rate of capital $\{R_b\}_{b=1}^{\mathcal{B}}$ for the year 2007 and other sector estimates. In the Supplemental Material we have additional estimation results that compare different estimates of ε_b . The results are similar to our baseline estimation with a one-period lagged instrument.

⁵The inclusion of fixed effects absorbs changes in the HHI that stem from the entry of more establishments.

⁶Local labor market summary statistics including quartiles of $HHI(s_{i|m})$ are in the Supplemental Material.

Table III1: Sector Estimates

| Sector Code | Sector Name | $\hat{\beta}_b$ | $\hat{\varepsilon}_b$ | $\hat{\varphi}_b$ | θ_b | R_b |
|-------------|-------------------------|-----------------|-----------------------|-------------------|------------|-------|
| 15 | Food | 0.74 | 1.69 | 0.22 | 0.13 | 0.11 |
| 17 | Textile | 0.74 | 1.49 | 0.51 | 0.02 | 0.14 |
| 18 | Clothing | 0.84 | 1.41 | 0.31 | 0.01 | 0.14 |
| 19 | Leather | 0.85 | 2.09 | 0.26 | 0.01 | 0.14 |
| 20 | Wood | 0.77 | 1.51 | 0.42 | 0.02 | 0.13 |
| 21 | Paper | 0.61 | 3.06 | 0.55 | 0.02 | 0.13 |
| 22 | Printing | 0.84 | 1.52 | 0.18 | 0.06 | 0.13 |
| 24 | Chemical | 0.67 | 3.25 | 0.06 | 0.14 | 0.16 |
| 25 | Plastic | 0.73 | 2.51 | 0.35 | 0.06 | 0.15 |
| 26 | Other Minerals | 0.65 | 1.62 | 0.43 | 0.05 | 0.15 |
| 27 | Metallurgy | 0.61 | 3.77 | 0.59 | 0.03 | 0.14 |
| 28 | Metals | 0.81 | 1.22 | 0.38 | 0.10 | 0.14 |
| 29 | Machines and Equipments | 0.79 | 2.18 | 0.32 | 0.09 | 0.17 |
| 30 | Office Machinery | 0.81 | 3.33 | 0.20 | 0.00 | 0.17 |
| 31 | Electrical Equipment | 0.65 | 3.02 | 0.67 | 0.04 | 0.23 |
| 32 | Telecommunications | 0.62 | 3.54 | 0.73 | 0.04 | 0.23 |
| 33 | Optical Equipment | 0.75 | 1.91 | 0.45 | 0.04 | 0.23 |
| 34 | Transport | 0.57 | 4.05 | 0.69 | 0.04 | 0.19 |
| 35 | Other Transport | 0.72 | 3.49 | 0.44 | 0.06 | 0.19 |
| 36 | Furniture | 0.81 | 1.57 | 0.43 | 0.03 | 0.14 |

Notes: $\hat{\beta}_b$: estimated output elasticities of labor, $\hat{\varepsilon}_b$: within local labor market elasticities, $\hat{\varphi}_b$: union bargaining powers, θ_b : intermediate good elasticities in the final good production function, R_b : capital rental rates following [Barkai \(2020\)](#).

IV Additional counterfactuals

We present additional results of the main counterfactual and other implications of labor market power on urban-rural differences.

IV.1 The effect of labor market power on urban-rural differences

Figure [IV2](#) suggests an important labor reallocation from cities to rural areas in the counterfactual without labor wedges. This section explores the impact of employer and union labor market power on the urban-rural mobility over time. We find that the importance of cities in manufacturing would have declined more slowly in absence of labor market power coming from firms and unions. A potential reason is that the closure of manufacturing establishments in cities would increase the labor concentration of urban areas, making small labor markets

Table III2: Estimated Within Elasticities for Different Lags

| Industry Code | Industry Name | 1 Lag $\hat{\epsilon}_b$ | 2 Lags $\hat{\epsilon}_b$ |
|---------------|-------------------------|--------------------------|---------------------------|
| 15 | Food | 1.69 | 1.99 |
| 17 | Textile | 1.49 | 1.83 |
| 18 | Clothing | 1.41 | 1.69 |
| 19 | Leather | 2.09 | 2.50 |
| 20 | Wood | 1.51 | 1.77 |
| 21 | Paper | 3.06 | 3.39 |
| 22 | Printing | 1.52 | 1.79 |
| 24 | Chemical | 3.25 | 3.56 |
| 25 | Plastic | 2.51 | 3.04 |
| 26 | Other Minerals | 1.62 | 1.77 |
| 27 | Metallurgy | 3.77 | 4.35 |
| 28 | Metals | 1.22 | 1.48 |
| 29 | Machines and Equipments | 2.18 | 2.63 |
| 30 | Office Machinery | 3.33 | 3.72 |
| 31 | Electrical Equipment | 3.02 | 3.61 |
| 32 | Telecommunications | 3.54 | 4.08 |
| 33 | Optical Equipment | 1.91 | 2.36 |
| 34 | Transport | 4.05 | 4.56 |
| 35 | Other Transport | 3.49 | 4.05 |
| 36 | Furniture | 1.57 | 1.90 |

Notes: All the estimated parameters are 2-digit industry specific. 1 Lag $\hat{\epsilon}_b$ are the estimated within local labor market elasticities when we instrument for the wages with one lag and 2 Lags $\hat{\epsilon}_b$ present the analogous when we instrument with two lags.

Table III3: Concentration and Labor Share

| | Dependent variable: $\log(LS_{h,t})$ | | |
|------------------------------|--------------------------------------|----------------------|----------------------|
| | (1) | (2) | (3) |
| $\log(\overline{HHI}_{h,t})$ | -0.064*** (0.013) | -0.054*** (0.013) | -0.056*** (0.014) |
| Sector FE | N | Y | N |
| Sector-year FE | N | N | Y |
| R ² | 0.017 | 0.290 | 0.343 |

Notes: The number of observations is 1,357. This table presents estimates of equation (?). Column 1 presents the estimate without any fixed effect. Column 2 shows results with sector fixed effects and column 3 has sector-year fixed effects. The dependent variable is the logarithm of 3-digit industry h labor share $\log(LS_{h,t})$ at time t . $\log(\overline{HHI}_{h,t})$ is the logarithm of the employment weighted average of the local labor market Herfindahl Index. *p<0.1; **p<0.05; ***p<0.01

relatively more attractive.

Employment changes

We compare the urban-rural manufacturing employment changes over time observed in the data to the ones from yearly counterfactuals without union and firm labor market power. In the data, the de-industrialization or the reduction of manufacturing employment occurred primarily in cities leading to the gain in relative importance of rural areas within manufacturing. Figure IV1 compares the relative employment shares observed in the data to the one in a counterfactual without labor wedges for each commuting zone.

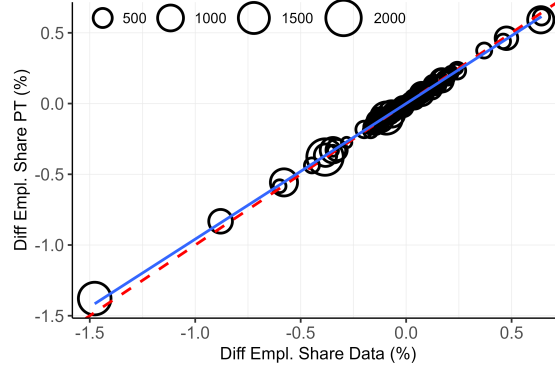
First, we perform the counterfactual where there are no labor wedges because establishments and unions act as price takers (PT) for the initial year 1994. Then we compute the commuting zone employment share out of total manufacturing employment for the initial and final years (1994 and 2007 respectively) and for the different scenarios. To compare mobility over time, we compute the differences over time of the commuting zone employment shares in the data ($\Delta^D = S_{07}^D - S_{94}^D$) and in the counterfactual ($\Delta^M = S_{07}^{PT} - S_{94}^{PT}$). Figure IV1 in presents this comparison. The x axis shows the time difference in the data Δ^D and the y axis shows the time difference in the model counterfactual without labor wedges Δ^M . The size of the dots is the initial level of manufacturing employment of the commuting zone. The counterfactual urban-rural mobility is very similar to the process observed in the data which is mostly guided by exogenous productivity and firm location decisions and not by labor market distortions.

The line generated by the largest population commuting zones in Figure IV1 is slightly flatter than the 45 degree line. Cities would lose their relative importance a bit slower in the counterfactual. A potential reason is the closure of manufacturing firms in the largest cities, which became more concentrated over time leading to distortions closer to the ones present in rural areas.

IV.2 Extensions

Table IV1 presents counterfactuals with agglomeration externalities under perfect competition where wages are equal to the marginal revenue product of labor. Baseline gains (1.62%) are amplified with agglomeration due to the productivity gains.

Figure IV1: De-industrialization differences



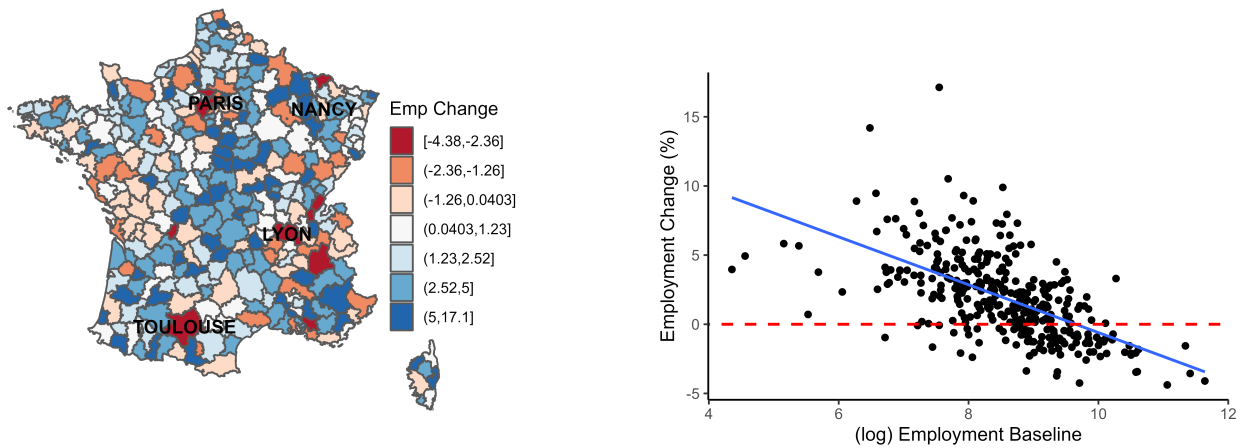
Notes: The x-axis shows the percentage differences of commuting zone employment shares out of manufacturing over time in the data ($\Delta^D = S_{07}^D - S_{94}^D$). The y-axis presents the analogous for the counterfactual without wedges ($\Delta^M = S_{07}^{PT} - S_{94}^{PT}$). The first year is 1994 and the last one is 2007. The bubble size represents the level of employment in thousands at the commuting zone for the first year. The blue line represents a fitted line from an OLS regression. A weighted least squares regression using initial employment as weights gives a similar result.

Table IV1: Counterfactuals: Agglomeration. Perfect Competition

| | ΔY (%) | $\Delta Prod$ (%) | Contribution ΔY (%) | | |
|-------------------------|----------------|-------------------|-----------------------------|--------------|-------|
| | | | GE | Productivity | Labor |
| <i>No Agglomeration</i> | 1.62 | 1.33 | 9 | 83 | 8 |
| <i>Agglomeration</i> | | | | | |
| $\gamma = 0.05$ | 1.73 | 1.40 | 8 | 82 | 10 |
| $\gamma = 0.1$ | 1.84 | 1.48 | 7 | 81 | 12 |
| $\gamma = 0.2$ | 2.08 | 1.66 | 5 | 80 | 15 |
| $\gamma = 0.25$ | 2.22 | 1.75 | 3 | 80 | 17 |
| $\gamma = 0.3$ | 2.36 | 1.86 | 2 | 80 | 18 |

Notes: Results are in percentages. First column ΔY is the change of aggregate output with respect to the baseline, $\Delta Prod$ is the change in aggregate productivity from decomposition (18). Last three columns present the contribution of each of the elements of the decomposition (18) to output gains. *No Agglomeration* is the main counterfactual without wedges, under free mobility of labor, fixed total labor supply and no agglomeration forces. All the other counterfactuals in this table allow for agglomeration within the local labor market. Similarly to the main counterfactual, workers are freely mobile and total employment is fixed. We present different counterfactuals depending on the agglomeration parameter γ .

Figure IV2: Employment Change (%) in the Counterfactual: Perfect Competition



Notes: The map presents employment changes with respect to the baseline economy in percentages within commuting zones. The counterfactual without labor wedges is performed for the year 2007. The figure in the right plots the employment change in the counterfactual versus the log of employment in the baseline. The blue line is a fitted line from an OLS regression.

Figure IV3 shows productivity changes in the counterfactual with oligopsonistic competition relative to the baseline. The map shows that the biggest productivity losses happen outside big cities and some commuting zones increase overall productivity due to labor mobility across sectors.

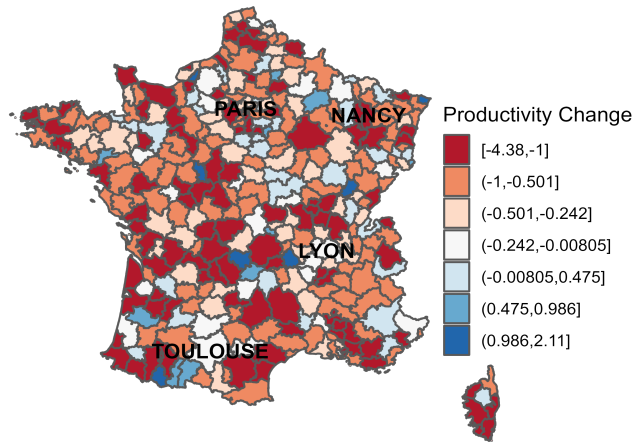
V Additional empirical evidence

We present an additional robustness exercise related to the reduced form regression in Section 2 in the main text, where we change the definition of the local labor markets, present results with an alternative instrument and the rent sharing results.

Local labor market. Figure VI does the same exercise as in the main empirical strategy but changing the definition of local labor market. Local labor markets are here defined with 2-digit industries instead of 3-digit industries.⁷ The specification includes commuting zone fixed effects as in Figure E1 Panel (b).

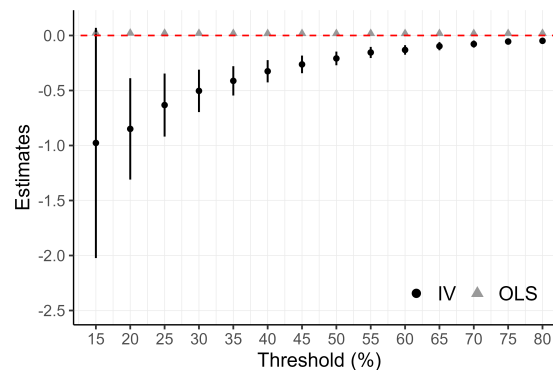
⁷That is, a local labor market is defined as a combination between commuting zone, 2-digit industry and occupation.

Figure IV3: Productivity Change (%) in the Counterfactual: Oligopsonistic Competition



Notes: The map presents productivity changes with respect to the baseline economy in percentages. Each block constitutes a commuting zone. Local labor markets are aggregated up to the commuting zone. Commuting zone productivity is an employment weighted average of individual productivities. Following the discussion in Section B.2, keeping fixed the baseline revenue productivities, any change in the counterfactual comes from changes in aggregate productivities from the reallocation of workers. Counterfactuals are performed for the year 2007.

Figure V1: Robustness. Local Labor Market at 2-digit Industry



Notes: This figure presents the point estimates and 95% confidence bands of the OLS and IV exercises on the y-axis. The x-axis presents different thresholds κ that define a mass layoff shock. We focus on non-affected competitors (not suffering a mass layoff shock). The instrument is the presence of a mass layoff shock firm in the local labor market. The definition of local labor market is a combination of commuting zone, 2-digit industry and occupation. The difference with respect to the figure in the main text is that the local labor market is at 2-digit rather than 3-digit industry.

Table V1: Wage Regression. Multilocation firm-occupations

| | <i>Dependent variable: $\log(w_{io,t})$</i> | | | |
|------------------|--|----------------------|---------------------|----------------------|
| | OLS | IV | OLS | IV |
| $s_{io m,t}$ | 0.010*** (0.001) | -0.030*** (0.002) | 0.007*** (0.001) | -0.030*** (0.002) |
| Firm-occ-year FE | Y | Y | Y | Y |
| CZ FE | Y | Y | N | N |
| CZ-year FE | N | N | Y | Y |
| Observations | 792,656 | 733,576 | 792,656 | 733,576 |
| R ² | 0.833 | 0.861 | 0.853 | 0.862 |

Notes: The instruments in this table are lagged concentration measures $1/N_{m,t-1}$. Columns 1 and 2 present estimates with commuting zone (CZ) fixed effects for the ordinary least squares (OLS) and instrumental variable (IV) exercises. Columns 3 and 4 present the analogous with commuting zone-year fixed effects. The dependent variable $\log(w_{io,t})$ is the logarithm of establishment-occupation wage at time t . $s_{io|m,t}$ is the establishment-occupation employment share at time t . * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Alternative instrument. Table V1 present results from an alternative instrument based on the lagged concentration measures. More specifically, we instrument the employment share $s_{io|m,t}$ by the lagged inverse of the number of competitors in the local labor market $1/N_{m,t-1}$. Lagged concentration measures exclude potential endogeneity of the market structure to current period shocks. The correlation between employment shares and lagged concentration measures is 0.77.

The first two columns recover estimates of the specification (1) with commuting zone (CZ) fixed effects and the last two columns with commuting zone-year fixed effects. Columns 1 and 3 present the ordinary least squares (OLS) estimates. The model reflects both labor demand and supply therefore a direct estimation by OLS is problematic and expected to be biased towards zero. We indeed find that both OLS estimates are very close to zero and positive. Columns 2 and 4 present the results once we instrument for the employment share. Both specifications (with CZ and CZ-year fixed effects) give the same point estimates. These estimates imply that an increase of one percentage point (p.p. henceforth) of the local labor market share is associated with a decrease of 0.03% of the plant wage. This implies that the same establishment passing from the first to the third quartile of the employment share distribution reduces wages by 0.68%. This elasticity translates into a reduction of roughly 190 euros of the median yearly establishment-occupation wage.

VI Data details

In this section we provide details about sample selection and variable construction.

VI.1 Sample selection

Ficus/Fare. This data source comes from tax records therefore we observe yearly firm information. We exclude the source tables belonging to public firms.⁸ Before 2000 we take table sources in euros and from 2001 onward we use data from consolidated economic units.⁹ After excluding firms without a firm identifier, the raw data sample contains about 29 million firms, of which about 2.8 million are manufacturing firms.¹⁰ Manufacturing sector (sector code equal to *D*) constitutes on average 10% of the observations, 19.2% of value added and 27.2% of employment.

Postes. *DADS Postes* covers all the employment spells of a salaried employee per year. If a worker has several spells in a year we would have multiple observations. The main benefit of this employer-employee data source is that we can know the establishment and employment location of the workers. We exclude workers in establishments with fictitious identifiers (SIREN starting by F) and in public firms. For every establishment identifier (SIRET) we sum the wage bill and the full time equivalent number of employees.

Merged data. After merging both data sources, we end up with data that include yearly establishment observations. After the filters and merging the sample consists of 1.3 million firms and 1.6 million establishment observations. In the process of filtering and merging, about half of the original firms are lost. Wages and value added are deflated using the Consumer Price Index.¹¹

Labor and wage data, coming from the balance sheets (at the firm level) and the one from employee records, needs to be consolidated. In order to be consistent with balance sheet infor-

⁸We only use the Financial units (*FIN*) and Other units (*TAB*) tables and exclude Public administration (*APU*).

⁹The profiling of big groups consolidates legal units into economic units. In 2001 the Peugeot-Citroën PSA was treated, Renault in 2003 and the group Accor in 2005. This implies the definition of new economic entities and would therefore lead to the creation of new firm identifiers. Given the potential impact of big establishments in local labor markets we opted to maintain them.

¹⁰We consider a missing firm identifier (SIREN) also if the identifier equals to zero for all the 9 digits.

¹¹Nominal variables are expressed in constant 2015 euros.

mation we assign labor and employment coming from *FICUS* to the establishments according to their respective shares. We proceed in several steps. First, we filter out observations with no wage or employment information from *Postes* from firms present at different commuting zones. Second, we get rid of observations with no labor, capital and wage bill information coming from *FICUS* and also observations with non existing or missing commuting zone. Third, we aggregate employee data to the firm times commuting zone level.¹² What we call establishment throughout the text is the entity aggregated at the commuting zone level. Then we compute the labor and wage shares of these entities out of the firm's aggregates. Finally, we split firm data from the balance sheet according to those shares. This procedure leaves the firms in a unique commuting zone with their balance sheet data but allows to split wage bill and employment data coming from the balance sheet for multi-location firms. Establishment wage is simply the average wage. That is, wage bill over total full time equivalent employees.

We further exclude Tobacco (2-digits industry code 16), Refineries & Nuclear industry (code 23) and Recycling (code 37). We finally get rid of the outliers reducing the sample 1.5% and finish with 4,156,754 establishment-occupation-year observations that belong to 1.25 million firms.¹³

VI.2 Variable construction

Ficus:

- Value added: value added net of taxes (*VACBF*). We restrict to firms with strictly positive value added.¹⁴
- Capital: tangible and intangible capital without counting depreciation. It is the sum of the variables *IMMOCOR* and *IMMOINC*.

¹²Data from 1994 and 1995 do not have commuting zone information. We therefore impute it using correspondence tables between city code and commuting zone. A city code has 5 digits coming from the department and city. Some commuting zone codes beyond the 2 missing years were modified or cleaned. City codes (*commune* codes) of Paris, Marseille and Lyon were divided into different *arrondissements*. We assign them codes 75056, 13055 and 69123 respectively. Then we proceed to the cleaning of commuting zones by assigning to the non existing codes the one corresponding to the city where the establishment is located. We get rid of non matched or missing commuting zone codes. We aggregate the data coming from *Postes* at the commuting zone level after this cleaning.

¹³We get rid of wage per capita outliers by truncating the sample at the 0.5% below and 99.5%.

¹⁴We follow the advise of the French statistical institute (INSEE) in using net value added to perform comparisons across industries.

- Employment: full time equivalent employment at the firm (*EFFSALM*).
- Wage bill: gross total wage bills. Is the sum of wages (*SALTRAI*) and firm taxed (*CHAR-SOC*).¹⁵
- Industry: industry classification comes from *APE*. The sub-industries *h* are 3 digit industries and industries *b* are at two digits.

Postes:

- Occupation: original occupation categories come from the two digit occupations (*CS2*). We group occupations with first digits 2 and 3 into a unique occupation group.¹⁶ This regrouping is done to avoid substantial changes in occupation groups between 1994 and 2007. Observations with missing occupation information are excluded.
- Employment: full time equivalent at the establishment-occupation level (*etp*).
- Wage: is the gross wage (per year) of individual worker (*sbrut*). If the spell is less than a year is the gross wage corresponding to the spell.
- Commuting zone: depending on the year, the commuting zone classification is taken from the variable *zemp* or *zempt*. Commuting zone information is missing for the years 1994 and 1995 and is imputed using the city codes.¹⁷

VI.3 Construction of required rates

In order to construct the required rates for the different sectors we follow the methodology proposed by [Barkai \(2020\)](#) using the Capital Input Data from the EU KLEMS database, December 2016 revision. In this dataset one can find, for a given industry, different depreciation rates and price indices for different types of capital. The types of capital that are present in the manufacturing sector are: Computing Equipment, Communications Equipment, Computer Software and Databases, Transport Equipment, Buildings and structures (non-residential), and Research

¹⁵For firms declaring at the BIC-BRN regime (*TYPIMPO*= 1) we only take *SALTRAI* .

¹⁶Occupations with first digit 1 and 7 are excluded. They constituted less than 0.05% of the matched sample.

¹⁷City codes are the concatenation of department (*DEP*) and city (*COM*).

and Development. We construct a required rate for each of the industries at the 2 digit level defined in the NAF classification. However, unlike the NAF classification, that we have up to 20 different industries, there are only 11 industries classified within manufacturing within the EU KLEMS database. Any industry classification in EU KLEMS is just an aggregation of the 2 digit industry classification in NAF. Therefore, there are industries within the NAF classification that will have the same required rate of return on capital.

For a type of capital s and sector b , we define the the required rate of return R_{sb} as:

$$R_{sb} = \left(i^D - \mathbb{E} [\pi_{sb}] + \delta_{sb} \right),$$

where i^D is a the cost fo debt borrowing in financial markets, and π_{sb} and δ_{sb} are, respectively, the inflation and depreciation rates of capital type s in sector b .

Then we define the total expenditures on capital type s in sector b as:

$$E_{sb} = R_{sb} P_{sb}^K K_{sb},$$

where $P_{sb}^K K_{sb}$ is the nominal value of capital stock of type s . Summing over all types of capital within a sector we can obtain the total expenditures of capital of such sector:

$$E_b = \sum_{sb} R_{sb} P_{sb}^K K_{sb}.$$

Multiplying and dividing by the total nominal value of capital stock we obtain:

$$\sum_s R_{sb} P_{sb}^K K_{sb} = \underbrace{\sum_s \frac{P_{sb}^K K_{sb}}{\sum_{s'} P_{s'b}^K K_{s'b}}}_{R_b} R_{sb} \underbrace{\sum_s P_{sb}^K K_{sb}}_{P^{Kb} K_b},$$

where the first term R_b is the interest rate that we use in the model.

VI.4 Other sources

The source to construct commuting zones from city codes is <https://www.insee.fr/fr/information/2114596> and the CPI data comes from <https://www.insee.fr/fr/statistiques/serie/001643154>.

Table VIII: Establishment-Occupation Summary Statistics

| | Mean | Pctl(25) | Median | Pctl(75) | St. Dev. |
|--------------------------|---------|----------|--------|----------|----------|
| All sample | | | | | |
| L_{iot} | 11.1 | 1.1 | 2.3 | 6.2 | 59.5 |
| $w_{iot}L_{iot}$ | 367.2 | 31.6 | 71.8 | 196.6 | 2,379.5 |
| w_{iot} | 34.0 | 20.9 | 27.4 | 39.5 | 117.1 |
| $s_{io m}$ | 0.20 | 0.01 | 0.05 | 0.24 | 0.30 |
| (a) Monolocation | | | | | |
| L_{iot} | 7.4 | 1.0 | 2.1 | 5.1 | 29.7 |
| $w_{iot}L_{iot}$ | 216.7 | 29.7 | 64.5 | 159.6 | 925.2 |
| w_{iot} | 32.8 | 20.3 | 26.6 | 38.5 | 35.5 |
| $s_{io m}$ | 0.18 | 0.01 | 0.04 | 0.19 | 0.29 |
| (b) Multilocation | | | | | |
| L_{iot} | 26.6 | 1.3 | 4.1 | 15.1 | 120.3 |
| $w_{iot}L_{iot}$ | 1,004.7 | 45.7 | 139.3 | 533.0 | 5,052.4 |
| w_{iot} | 39.0 | 23.6 | 30.7 | 43.7 | 257.7 |
| $s_{io m}$ | 0.29 | 0.02 | 0.11 | 0.48 | 0.35 |

Notes: The top panel shows summary statistics for the whole sample. Panels (a) and (b) present respectively summary statistics of monolocation and multilocation firm-occupations. Number of observations for All Sample is 4,151,892. The Monolocation sample is 3,359,236; and the Multilocation sample is 792,656. L_{iot} is full time equivalent employment at the establishment-occupation io , $w_{iot}L_{iot}$ is the wage bill, w_{iot} is establishment-occupation wage or wage per FTE, $s_{io|m}$ is the employment share out of the local labor market. All the nominal variables are in thousands of constant 2015 euros.

VII Summary statistics

Tables VIII1, VIII2, VIII3 and VIII4 contain summary statistics of establishment-occupations, 3-digit industries, local labor markets and commuting zones for the year 2007, which is the year we use for our counterfactuals. Table VIII5 presents worker transition probabilities across occupations, industries and commuting zones.

Table VII2: Sub-industry Summary Statistics. Baseline Year

| Variable | Mean | Pctl(25) | Median | Pctl(75) | St. Dev. |
|-------------|--------|----------|--------|----------|----------|
| N_h | 2,840 | 493 | 1,261 | 2,639 | 4,530.5 |
| L_h | 30,466 | 7,559 | 15,070 | 50,036 | 33,899.3 |
| \bar{w}_h | 34.6 | 29.6 | 33.0 | 37.531 | 6.9 |
| LS_h | 0.52 | 0.48 | 0.53 | 0.58 | 0.10 |
| KS_h | 0.26 | 0.17 | 0.23 | 0.32 | 0.13 |

Notes: There are 97 3-digit industries, or sub-industries, in the sample. N_h is the number of establishments per 3-digit industry h , L_h is total employment of h , \bar{w}_h is the average establishment wage of h , LS_h is the labor share and KS_h is the capital share. We get the capital shares following Barkai (2020). All the nominal variables are in thousands of constant 2015 euros.

Table VII3: Local Labor Market Summary Statistics. Baseline Year

| Variable | Mean | Pctl(25) | Median | Pctl(75) | St. Dev. |
|--------------------------|------|----------|--------|----------|----------|
| N_m | 4.76 | 1 | 2 | 4 | 14.4 |
| L_m | 51.0 | 2.8 | 9.4 | 34.9 | 196.2 |
| \bar{w}_m | 36.6 | 24.3 | 30.2 | 42.5 | 36.1 |
| \hat{w}_m | 36.2 | 24.1 | 30.0 | 42.2 | 25.6 |
| $\text{HHI}(s_{io m})$ | 0.67 | 0.38 | 0.68 | 1.00 | 0.32 |
| $\text{HHI}(s_{io m}^w)$ | 0.68 | 0.39 | 0.70 | 1.00 | 0.32 |

Notes: There are 57,940 local labor markets in the year 2007. N_m is the number of competitors in the local labor market m , L_m is total employment in m , \bar{w}_m is the mean w_{iot} of the establishment-occupations in m , \hat{w}_m is the weighted average wage at m with weights equal to employment shares, $\text{HHI}(s_{io|m})$ and $\text{HHI}(s_{io|m}^w)$ are respectively the Herfindahls with employment and wage shares. All the nominal variables are in thousands of constant 2015 euros.

Table VII4: Commuting Zones Summary Statistics. Baseline Year

| Variable | Mean | Pctl(25) | Median | Pctl(75) | St. Dev. |
|-------------|-----------|-----------|-----------|------------|------------|
| N_n | 773.798 | 266.8 | 461 | 861.2 | 1,168.407 |
| L_n | 8,300.567 | 2,567.403 | 5,244.300 | 10,086.210 | 11,322.000 |
| \bar{L}_n | 11.389 | 8.148 | 10.878 | 13.547 | 6.043 |
| \bar{w}_n | 34.399 | 32.707 | 34.161 | 35.593 | 3.242 |

Notes: There are 356 commuting zones in the sample. N_n is the number of establishments at the CZ, L_n is full time equivalent employment at CZ, \bar{L}_n is the average L_{iot} of establishment-occupations at n , \bar{w}_n is the mean w_{iot} at n in thousands of constant 2015 euros.

Table VII5: Transition Probabilities

| Occupation | Commuting Zone | Industry | Trans. Prob. FTE | Trans. Prob. |
|------------|----------------|----------|------------------|--------------|
| 0 | 0 | 0 | 91.39 | 91.01 |
| 0 | 0 | 1 | 2.37 | 2.36 |
| 0 | 1 | 0 | 0.02 | 0.02 |
| 1 | 0 | 0 | 6.03 | 6.40 |
| 1 | 0 | 1 | 0.20 | 0.21 |
| 1 | 1 | 0 | 0.00 | 0.00 |
| 1 | 1 | 1 | 0.00 | 0.00 |

Notes: The transition rates are computed over the whole sample period 1994-2007. *Occupation* is an indicator function of occupational change, *Commuting Zone* is an indicator function of commuting zone change, *Industry* is an indicator function of 3-digit industry change, *Trans. Prob. FTE* are the unconditional transition probabilities based on full time equivalent units and *Trans. Prob.* are the unconditional transition probabilities based on counts of working spells independently of duration and part-time status.

Table VIII1: Rent Sharing: Industry

| Code | Ind. Name | Estimate | SE ($\times 10^2$) | Code | Ind. Name | Estimate | SE ($\times 10^2$) |
|------|-----------|----------|----------------------|------|----------------------|----------|----------------------|
| 15 | Food | 0.40 | 0.09 | 24 | Chemical | 0.17 | 0.17 |
| 17 | Textile | 0.22 | 0.23 | 25 | Plastic | 0.23 | 0.21 |
| 18 | Clothing | 0.31 | 0.18 | 26 | Other Minerals | 0.25 | 0.18 |
| 19 | Leather | 0.31 | 0.39 | 27 | Metallurgy | 0.14 | 0.40 |
| 20 | Wood | 0.32 | 0.24 | 28 | Metals | 0.37 | 0.12 |
| 21 | Paper | 0.22 | 0.37 | 29 | Machines and Equip. | 0.30 | 0.14 |
| 22 | Printing | 0.34 | 0.11 | 30 | Office Machinery | 0.33 | 0.56 |
| | | | | 31 | Electrical Equipment | 0.25 | 0.23 |

VIII Additional reduced form exercise: rents and wages

Workers organize into unions to extract rents from the firm through bargaining. Bargaining can happen at different levels in France and here we want to inform the modeling decisions by quantifying bargaining differences depending on industries or occupations. We build a proxy of rents at the firm level and then compare how the correlation of wages with rents is differentiated depending on the industries and occupations. In particular we compute rents at the firm level $y_{J(i),t}$ by computing value added minus capital expenditures per worker. The reduced form model is:

$$\ln w_{io,t} = \gamma_k \ln y_{J(i),t} + \epsilon_{io,t},$$

where γ_k is the elasticity of wages with respect to rents and k denotes either 2-digit sector b or occupation o , $y_{J(i),t}$ is the proxy of rents at the firm level and $\epsilon_{io,t}$ is the error term.

Tables VIII1 and VIII2 present respectively the estimated rent sharing elasticities for industries and occupations. As it is clear from comparing the tables, there is more heterogeneity in the rent sharing elasticities across industries than across occupations. Given the higher dispersion of the elasticities at the sector level, we assume differentiated bargaining powers depending on the sector in the model.

Table VIII2: Rent Sharing: Occupation

| Occupation | Estimate | SE ($\times 10^2$) |
|----------------|----------|----------------------|
| Top management | 0.38 | 0.08 |
| Supervisor | 0.27 | 0.06 |
| Clerical | 0.29 | 0.06 |
| Blue collar | 0.30 | 0.05 |

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